a journal of the Society for Music Theory

Volume 8, Number 3, October 2002

Transformational Networks, Transpositional Combination, and Aggregate Partitions in *Processional* by George Crumb

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KEYWORDS: Crumb, transformations, transpositional combination, aggregate partition, networks, transpositionally invariant sets

ABSTRACT: George Crumb is popularly known as a composer who employs extended instrumental techniques. This characterization could divert attention away from investigating pitch structures in his compositions. Although some theoretical works have begun focusing attention on pitch, the scope of these investigations has been limited to the procedures associated with a limited number of symmetrical sets. Focusing solely on symmetrical sets could overshadow other methods of organizing pitch. Through an analysis of the solo piano work *Processional* (1983), I will demonstrate that Crumb's procedures include techniques that link the compositional opportunities symmetrical sets offer to the procedures associated with aggregate-based atonal composition. The analysis will reveal that symmetrical and non-symmetrical set structures in *Processional* are part of a larger group of relations that include techniques such as aggregate partitions, transpositional combination, and transformational networks. My analysis will also demonstrate how these techniques and the techniques associated with symmetrical sets blend to create a larger compositional universe. Finally, I will suggest a more general model for the various networks that appear in *Processional*.

Received 28 April 2002

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[1] The propensity of some composers to gravitate towards a select group of set classes as the source of musical structure in their works has become a type of equivalence relation in recent theoretical writings that tends to group some composers into two large classes. Richard Bass has noted that this bifurcated view of pitch structure essentially places Schoenberg and his followers on one side of the divide and on the other side are composers, such as Bartok, Stravinsky, and Messiaen, whose compositional procedures are inextricably linked to symmetrical set classes.(1) Another factor contributing to this bifurcated view is the tendency to raise the status of symmetrical set classes to be on par with but independent of the diatonic collection. That is, symmetrical set classes are seen as functionally equivalent to their diatonic counterparts in their capacity to function as referential collections that generate musical structure. They are emancipated from the diatonic, because the structures and procedures they produce do not need to be legitimized as originating with the diatonic. The simultaneous functional association of symmetrical set classes with the diatonic and functional emancipation of symmetrical set classes from the diatonic widens the gap between the classes of composers, since the compositional procedures of Schoenberg and his followers tend not to be seen through the same referential filter. A passage from the conclusion of Bass's article implicitly suggests that this might be the case:

> The octatonic and whole-tone elements in "Music of Shadows,". . .are distinctive in their emancipation from any enlarged diatonic context. At the same time, labels such as "chromatic" or "atonal" are too general to account for the pitch-structural orientation of the piece. The interpenetration of these referential collections is not without precedent, but Crumb's specific approach is unique in its elevation of existing techniques to the level of independent procedures capable of generating motivically unified, complete musical structures. . . The ascendancy of aggregate-based atonal and serial methods during the mid-twentieth century may have temporarily relegated these symmetrical referential collections to a subordinate role, but recent works by a number of composers provide evidence that the compositional opportunities offered through interaction between octatonic, whole-tone, and related sonorities. . .were not exhausted in the early part of the century. Crumb in particular has developed clear and aurally accessible models for the integration of two, and sometimes more, non-diatonic reference sets that stand on a par with diatonic and chromatic writing within the broader spectrum of his eclectic harmonic language. (2)

[2] Richard Cohn expounds a similar but more general view of symmetrical set classes in his article "Properties and Generability of Transpositionally Invariant Sets." (3) As Cohn's article clearly states and Bass' article implicitly states, the select group of set classes that composers gravitate towards is the collection of transpositionally invariant or transpositionally symmetrical set classes (see **Example 1a**). Cohn claims the gravitational pull of this collection of set classes for a composer lies in their modes of generability. This means each member of this select group is capable of being generated by a multitude of transformations of its transpositionally related subsets, and conversely, each member of this group can be disunited into a multitude of transpositionally related subsets. **Example 1b**, adapted from Cohn's article, illustrates that members from each of the set classes 2-2, 2-4, 3-8, and 4-25, which are all members of Cohn's cyclic homomorphic equivalence class [02] mod 6, generate a member of set class 4-25 under the operation of transpositional combination with T_6 as the operand. Conversely, members of set class 4-25 can be disunited into T_6 related members of the generating set classes.

[3] Modes of generability is the fundamental transformational process that, according to Cohn, reduces the power of diatonic interaction as an explanation for the popularity of the TINV family of sets. One might suppose that if the ability to interact with the diatonic generated the utility of TINV sets classes, then both collections would share some essential structural features. For example, besides transpositional invariance, TINV set classes are also inversionally invariant, a property TINV set classes share with the diatonic hexachord 6-32 [024579]. Cohn notes, however, that TINV and diatonic collections rate below average on any number of similarity scales. He goes on to say that dissimilarity is not, of course, a prophylactic against TINV and diatonic interactions, nor is it a preventative against raising those interactions to the level of forming a compositional syntax, but the dissimilarity of the collections suggests that a special relationship between the collections is not the source of their interaction. Cohn cites historical/semantic reasons for why TINV collections might want to legitimize themselves by associating with the more established member of the diatonic, but "given a diatonic collection as a compositional premise, it is not yet clear why TINV collections should be chosen as playmates." (4)

[4] One aim of this study is to build a little more of one particular bridge across the canyon separating the two views of pitch structure. Cohn laid this bridge's foundation by noting how modes of generability are related to research in combinatoriality. The material for our bridge's roadway will come from the diatonic collection of sets. Consequently, another aim of this paper is to explore syntactic connections that make the diatonic and TINV collections good playmates. We will explore these issues by means of an analysis of Crumb's solo piano work *Processional* (1983). I will demonstrate that the work's compositional procedures include techniques that link the opportunities TINV collections offer to procedures associated with aggregate-based composition. The analysis will also reveal that TINV and non-TINV set structures in *Processional* are part of a larger group of relations that include techniques such as aggregate partitions, transpositional combination, inversional symmetry, generalized CUP relations, K-nets, and other transformational networks. (5)

[5] Before we begin, however, we need to establish a rule of the game. "Aggregate-based composition" is a loaded expression that means many things to many people. One association I would like to avoid is that aggregate-based composition implies perceiving aggregate completion as a necessary foundation for comparing aggregates and their contents. I don't want to avoid the sense of aggregate completion requiring the presence of all twelve tones, but I would like to avoid the implication of the aggregate as a perceptual unit that once perceived signals a change from one aggregate to another. This form of aggregate completion is most often associated with some forms of serial composition. In some serial contexts, we know we have moved from one row form to another or from one aggregate to another, because hearing the completion of an aggregate marks the boundary between aggregates. We can discover the transformational relationship between two row forms or compare the configuration of transformationally related subsets within each aggregate, once the aggregate

boundary has been perceived. Most often, the non-immediate repetition of a pitch-class determines or can act as a signal that the aggregate boundary has been reached. Since *Processional* uses unordered or non-serially ordered collections, pitch-class repetition is an ineffective marker of aggregate boundaries, because the repetition of a pitch-class does not signify aggregate completion. Furthermore, any number of events can signal or be the impetus for the change from one aggregate to the next. Aggregates in the present discussion are, to use Robert Morris' term, compositional spaces that are contained within the larger compositional space produced by the transformational networks that link aggregates. In *Processional,* the completion of one process and the initiation of a new process often determine the change from one aggregate to the next. In the former view, aggregate perception is necessary and it forms the basis of local comparison, while in the latter view local comparison does not depend on perceiving aggregates. I would also like to extend the concept of an aggregate compositional space to include transformations as well pitch classes. Object and process are inextricably intertwined in *Processional,* so aggregate transformational structures often mirror aggregate pitch-class structures. Furthermore, the concept of an aggregate transformational space is one source of the work's coherence, since the space is closed with regard to the particular transformation. Partitioning the aggregate transformational space generates many of the work's formal divisions. As I will demonstrate, these surface formations are the result of deeper or hidden processes.

[6] *Processional,* as a score and composition, has several features that are worth noting. The score does not contain any barlines, except for double bars that mark the work's large divisions. In many ways, the absence of barlines is a visual indicator that underscores viewing the piece as a single unfolding process, such as the process that takes a fertilized egg from single cell to a multi-celled functioning organism. The score also lacks any of the usual timbral devices, such a plucking the strings inside the piano, which are associated with Crumb's music. The lack of extended timbral techniques focuses attention on the work's pitch-class structures. Crumb does include a one page appendix containing six *Ossia* passages with, in the composer's words, "a few extended piano effects," perhaps for players who may miss this aspect of his music. Although the work lacks barlines and it is a single unfolding process, it does divide up into several sub-processes (see **Example 2**). The work contains three large sections at the global level in the familiar pattern A-B-A. The presence or absence of key signatures and the double bars distinguish the A and B sections. The A sections each contain four sub-sections that are distinguished by changing key signatures. Although the B section lacks divisions marked by change of key, it also contains four sub-sections marked by change of texture. Sections one and four are homophonic, section two is contrapuntal, and section 3 is a hybrid homophonic/contrapuntal texture.

[7] *Processional* begins with a descending six-note motive that is a member of the set-class 6-32[024579] commonly known as the diatonic hexachord (**Example 3a**). Moving quickly to a high level of generalization, however, bypasses many of the hexachord's more important features and implications generated by its pitch realization. The three semitone or interval class (hereafter ic) 3 "gap" at the hexachord's center splits it into two trichordal subsets that are members of set class 3-6[024]. Although many other pitch realizations, such as transposing the lower trichord up an octave (**Example 3b**), produce similar results, the ic 3 gap at the hexachord's center perhaps emphasizes both the independence of the 3-6[024] trichords and their role in generating the larger set. The "octave up" pitch realization, for example, may at a higher level of abstraction contain the same information, but it is not quite set into the same relief, since it perhaps emphasizes the hexachord and de-emphasizes the trichords. While the pitch realization of the motive emphasizes the generative role 3-6[024] trichords may play in producing the hexachord, it does not say anything definitive about the chosen generative path. Perhaps the simplest transformational route would be by means of transpositional combination (hereafter referred to as TC) of the lower trichord at T_7 (see **Example 3c**). However, the arrangement of the pitches around the gap also strongly suggests inversional symmetry at T_eI or inversional symmetry around the pitch dyad B3/C4, or more generally IC/B, as alternate transformational routes.(6)

[8] The inversional dyadic center plays another important role with regard to another ambiguity created by this motive's pitch realization. The subset-superset relation created by the trichord partitioning of the hexachord suggests inclusion relations may play an important role in this work (**Example 4**). For example, including either member of the dual axis of symmetry around which the 6-32 [024579] hexachord is constructed generates two different members of the set class 7-35[013568t]. While the "key signature" of the piece implies a 7-35[013568t] superset with a pitch class content of {D♭, E♭, F, G♭, A♭, B♭, C} for the 6-32[024579] hexachord, the pitch realization of the motive and the choice of either pitch class B or C to fill the gap strongly suggests the pitch class content of the member of the 7-35[013568t] set class could also be {G♭, A♭, B♭, C♭, D♭, E♭, F}. We will return to the role the ambiguity plays in the opening section of the work shortly. Combining both members of the dual axis of symmetry with the 6-32[024579] hexachord constructed around the axis produces a member of set class 8-23 [0123578t] whose complement is 4-23[0257], a tetrachord important to the B section of the work.

[9] The trichord partitioning of the hexachord also suggests hexachords may be part of a larger partitioning scheme. Each of the outlined transformational routes, for example, determines a different partitional path that the motive may travel in its development (**Example 5a**). Continuing on the TC path produces a series of T_7 related 3-6[024] trichords. The T_7 path that generates trichords is suggestive, because T_7 cycles of pitch classes generate the collections 6-32[024579], 7-35[013568t], and 8-23 [0123578t]. Of course, the new trichords maintain all the structural features of the generating pair, but some of those features are lost at the level of the hexachord. For example, unlike the trichords, the hexachords do not maintain the non-intersecting pitch-class content feature of the trichords. Continuing on the path of inversional symmetry, however, produces a new pair of trichords that maintain the structural features of the generating pair, and it produces a new hexachord that maintains the non-intersecting pitch-class content feature of the trichords with the original hexachord (see **Example 5b**). The two diatonic hexachords, of course, produce the aggregate, since their pitch-class content is non-intersecting or complementary with regard to the total chromatic. If, however, the inversional center partitions the aggregate rather than the diatonic hexachord, then the two hexachords generated by the transformational schema are members of set class 6-35[02468T] or the whole tone collection. As we shall see shortly, other combinations or partitions of this aggregate's components produce other hexachordal profiles. Therefore, perhaps rather than viewing any particular hexachord as being fundamental, the 3-6[024] trichord and its transformational stance should be thought of as fundamental, in the same way that plate tectonics is responsible for the surface formations of the earth. That is, surface formations are the result of deeper or hidden processes.

[10] The question now, of course, is how does this transformational model play out in the music (see **Example 6**). While the D♭ 6-32 hexachord settles into an ostinato, the pitch classes, 11, 7, 4, 9, 2, and 0 gradually enter in the registers surrounding the D♭ 6-32 hexachord producing its complement at T_6. If, for the moment, we view the pitches outside the range of the model, which is C3 through B♭3 and D♭4 through B4, as a third registral stream, we see that the new pitch material with one very important exception completes the inversional transformational schema begun by the D♭ 6-32 hexachord.[(7)] The C3 of the model is the exception, since it always appears as C4, one of the pitches from the inversional center. The deviation from the model is the result of two special functions pitch class 0 performs in this section. It is one of two pitch classes that can deny or confirm the implied seven-note collection of the key signature. It is also the pitch class that completes the aggregate. Both of these functions come into play at the end of the first section.

[11] Although the C4 completes the pitch-class aggregate in the third system of Example 6, the action within this aggregate compositional space continues, because the process behind the procession in this section of *Processional* has not reached its completion. Prior to this point, the ostinato hexachord has not remained unaffected by the procession of notes around it. It constantly reinvents its set-class profile by losing its own members, acquiring members from its complement, or both losing and acquiring new members. Although the full implications of how these changes contribute to the structure of the work are beyond the scope of this paper, we can examine one or two key relationships. For example, with the first appearance of B2 in the outer stream, the G♭3 disappears from the center stream (see Example 6). The pitch-class exchange produces a new hexachordal set class, 6-33 [023579]. Assuming for a moment a more tonally oriented hearing, the change of hexachord could be heard as a "shift" from a major to minor sound. The G4 entrance following the appearance of B2 replaces B2, but it maintains the hexachordal set class 6-33 [023579]. The appearance of E3 two eighth notes later is marked by the reappearance of G♭3 generating the work's first seven-note collection, a member of set class 7-23 [0234579]. The appearance of 7-23 marks the end of the work's first sub-process, since it fuses or unites the 6-32 and 6-33 hexachords into a single seven-note collection that subsumes both hexachords. The music needs to continue, however, because the larger process, which is related to the appearance of 7-23, has not reached it conclusion. That is, the appearance of 7-23 does nothing to disambiguate which member of the 7-35 set class the D♭ 6-35 hexachord will become, D♭ 7-35 or G♭ 7-35.

[12] When the aggregate is finally completed with the entrance of the C4, the process of hexachordal reinvention continues. For example, the C4 entrance produces a 6-Z28 [013569] with four members of the opening 6-32 hexachord {D♭, E♭, F, and G♭} along with A4 from the opening hexachord's complement.[(8)] Once C4 is introduced, it remains, for the most part, a member of the pitch sets formed by the ostinato (**Example 7**). Although the ostinato process continues to produce members of various set classes, it eventually settles on the member of 7-35 that confirms the key signature, {D♭, E♭, F, G♭, A♭, B♭, C}. C4 is the only pitch from the complementary hexachord that is allowed to invade the registral space of the D♭ 6-32 hexachord, and the union of C4 with D♭ 6-32, not the completion of the aggregate, is the event that brings the section to its end. The completion of the aggregate does, however, play the role of triggering the event that does bring the section to its close.

[13] Although the inversional transformational schema models the pitch realization of the 3-6[024] trichords, a more general

pitch-class transpositional network based on TC relations produces another model of the four 3-6[024] trichords that better explains their relationship to other structural features of the composition. Taking the pc set {024} as the point of origin or T_0, the remaining trichords relate to it by T_1, T_6, and T_7 (**Example 8a**). In this model, combinations of interval cycle 6 or C6, following Perle and Cohn, generate the four trichords.(9) An interesting consequence of this generative process is that it has two faces that are revealed by exchanging object and process in the matrix. If the transformational process, T_0, T_1, T_6, T_7 becomes the pitch class set {0167}, and the pitch class set {024} becomes the transformational process T_0, T_2, and T_4, the new process generates a differently partitioned aggregate from members of set class 4-9[0167]. Applying the $T_{(024)}$ process to the original matrix produces an interesting and related result. **Example 8b** illustrates, that the three matrices produce all twelve transpositions of the 3-6[024] trichord, which represents another level of saturation. The exchange of process and object in this aggregate generating context creates a bond between two set classes that do not rate very highly on any of the conventional similarity scales, and it bonds a member of the TINV collection with a set class from outside its world.

[14] The $T_{(0167)}$ transformation is not the only cyclic generator of the aggregate bonded to the 3-6[024] trichord. When the pitch C4 completed the aggregate, it did so against a 6-Z28 [013569] hexachordal backdrop. This hexachord weaves together the {D♭, E♭, F} 3-6 trichord together with a member of set class 4-28[0369], a set class whose interval structure links it to the interval class 3 that separates the D♭ 6-32's 3-6 trichords and the $T_{(0167)}$ transformation that generates the 3-6[024] trichords of the first section. As **Example 9** illustrates, translating 4-28 from object to process and applying the process to the pitch class set {024} produces another partitioning of the aggregate using 3-6[024] trichords. The $T_{(0369)}$ matrix pairs two trichords from the $T_{(0167)}$ with a new pair. Applying T_1 and T_2 to the $T_{(0369)}$ matrix produces two new matrices that in combination with the first matrix produce all twelve members of set class 3-6[024] (Example 9b). All of the trichordal relationships can be extended to the hexachordal level by applying the same TC operation that produced the D♭ 6-32 hexachord from the 3-6 trichords to each of the $T_{(0369)}$ matrices. For example, combining the T_1 matrix with T_5 of itself produces a hexachordal matrix containing the T_6 related 6-32[024579] hexachords from the first section (see **Example 10a**). Applying the same process to the T_0 and T_2 matrices produces two new matrices that in combination with the first matrix produce all twelve members of set class 6-32[024579] (see **Example 10b**). As Example 10b illustrates the columns containing identical trichords can function as bridges facilitating movement from one matrix to another. The type of saturation exemplified by the complex of matrices plays a critical role in the architecture of *Processional.*

[15] The work's second section, demarcated in the score by the change of key signature from five flats to four sharps and demarcated in the processional by another process, continues the homophonic texture from the end of the previous section (see **Example 11**). Although the new section continues the previous section's texture, the processes generating musical development begin all over again. The first chord of section two, E 6-32[024579], repeats the generative role played by the D♭ 6-32[024579] hexachord in section one. Its two constituent 3-6[024] trichords are arranged in pitch space to produce the 3 semitone or ic 3 gap, for example. In fact, the entire structural complex of relationships attributed to D♭ 6-32[024579] apply to E 6-32[024579], since they are T_3 transformations of each other in both pitch class and pitch. The transpositional relationships shared by the generative hexachords in both sections imply that the inversional transformational model of section one might likewise be transposed and at work in section two. As example 11 illustrates, the notes of the complementary hexachord, {G, D, F, B♭, C, D♯}, quickly enter and complete the aggregate space. If, for the moment, we ignore the pitches outside the center stream range of E♭3 through C♯3 and E4 through D5, we see that the new pitch material with one very important exception produces a T_3 transposition of the model in pitch class and pitch.

[16] The one exception is D♯3, and once again it is one of a pair of pitch classes that can confirm or deny the 7-35[013568t] collection implied by the key signature. Pitch class 3's special status is once again confirmed by one of its representatives, D♯3, since D♯3 is the only pitch allowed to invade the registral space of the E 6-32[024579] hexachord. In section two, however, the collection confirming pitch class takes on a more subversive role by revealing that its allegiance may actually be with the other hexachordal set class of the model, the whole tone collection or 6-35[02468t]. Section one ended with a stabilization of D♭ 7-35 produced by C4 forming a union with D♭ 6-32. Section two ends with a rising whole tone collection beginning on G3 and ending on D♯4 that undermines any attempt of the chords to establish an E 7-35[013568t] collection (see Example 11). The significance of this development will shortly become evident.

[17] Sections one and two present several musical streams, the homophonic texture of the middle register and the single note events that unfold around the center form two streams.(10) Each stream unfolds complementary related hexachords. Focusing on the center stream for a moment reveals that its transpositional process is directly related to the T_1 $T_{(0369)}$ matrix of Example 10b, since the first two hexachords of the matrix are the hexachords unfolded in the center stream. The

outer registral stream simply follows a rotated version of the matrix related to the original matrix by T_6. These relationships are more easily graphed by collapsing each hexachord into its first pitch class and taking that pitch class as a representative of the hexachord and as a representative of the member of the cycle generating the matrix (see **Example 12**). In the example, the number 1 represents the D♭ 7-32[024579] hexachord and the 1 of the 1-4-7-t (0369) cycle that generates the T_1 $T_{(0369)}$ matrix. While Example 12 is a graph of the actual streams of sections one and two, it is also a projection of how the streams might continue. Both the T_1 matrix and each stream of the graph contain a natural partition that divides each object into two parts. The hexachords D♭ 6-32[024579] and E 6-32[024579] each share three common tones and are on one side of the divide, while their complements, G 6-32[024579] and B♭ 6-32[024579] also share three common tones and are on the other side of the divide. The graph of Example 12 reveals another perspective on the relationship of complementary hexachords. In the second half of the graph, the hexachords from the outside stream move inside and vice versa.

[18] While each half of the T_1 T(0369) matrix and each half the graph contain complementary hexachords, adjacent hexachords in the T_1 T(0369) matrix and in the individual streams of the graph share three common tones. Furthermore, the non-common tones are all adjacent pitch classes. As the joint between sections one and two illustrates, these pitch class properties translate into extremely smooth or parsimonious voice leading in the pitch dimension (see **Example 13**). The pitches of the D♭ 6-32[024579] hexachord connect to the pitches of the E 6-32[024579] hexachord by common tone or half step motion. Common tones link the outer stream as well, but in a different manner. In section one, two pitches, B2 and A2, that are members of a third registral stream but were in essence temporarily relegated to the role of octave duplications of the inversional transformational model's G 6-32[024579] hexachord, play an important role in linking sections one and two. These pitches are members of the {G, A, B} 3-6[024] trichord that forms the upper trichord of the model in Example 5. Although pitch G2 does not appear in section one, it is a common tone shared by G 6-32[024579] and the B♭ 6-32[024579] hexachord in the outer registral stream, and it is the first pitch to appear in the outer stream in section two creating a long range {G, A, B} 3-6[024] trichord connecting sections one and two and shedding new light on the generative set class of the work.

[19] If the T_1 $T_{(0369)}$ matrix model of hexachordal progression is the model or process directing hexachordal motion in the work, then the next hexachord to appear in the center stream should be G 6-32[024579], the hexachord from section one's outer stream. As the key signature change in score indicates (see **Example 14**), the G whole tone collection ending on D♯ leads directly to the G 6-32[024579] hexachord. As was the case previously, the entire structural complex of relationships attributed to D♭ 6-32[024579] and E 6-32[024579] apply to G 6-32[024579], since they are T_6 and T_3 transformations, respectively, of each other in both pitch class and pitch. As was demonstrated with the graph of Example 12, the arrival of G 6-32[024579] is an important event, since it marks the first repetition of a hexachordal collection, it is the first time hexachords have crossed in the inner streams, and it marks the halfway point in the movement through (0369) cycle governing hexachordal progression. With its T_6 related 3-6[024] trichords and its three interval-class 6s, preceding the hexachordal change with the whole tone collection underscores the significance of the change.

[20] Although the T_1 $T_{(0369)}$ matrix model of hexachordal progression predicts the next hexachordal change in the center stream (the hexachordal stream that determines the key signature) should be a member of B♭ 6-32[024579], the hexachord that arrives is B 6-32[024579] (see Example 14). The new hexachord is a member of the T_2 $T_{(0369)}$ matrix. Even though the expected hexachord does not arrive, the interloper, as if trying to go unnoticed, continues the musical processes that characterized sections one through three. The B 6-32[024579] maintains the pitch realization of its T_7 related 3-6[024] subsets, including the three semitone or ic 3 gap. The pitch classes of its T_6 complement enter in pretty much the same fashion as in previous section. In the final chord of the section, the pitch A♯4 fills in the ic 3 gap and stabilizes the seven-note collection as B 7-35[013568t]. Finally, the inversional transformational model underlying the pitch realization of both hexachordal streams in section one through three is also at work here as well.

[21] Two non-mutually exclusive explanations can account for this departure from the T_1 $T_{(0369)}$ matrix model of the process governing hexachordal progression in *Processional*. First, since the previous section saw the first return of a hexachordal collection and the crossing of hexachordal streams, section three was the beginning of a modified return that sufficiently represented the model, so there is no need to continue along its outlined path. Consequently, the music is free to develop in other directions. Second, the B 6-32[024579] hexachord really could be an interloper that just temporarily halts the progression of the T_1 $T_{(0369)}$ hexachordal progression. In the latter case a new stream begins before the active stream finishes implying that both streams, T_1 $T_{(0369)}$ and T_2 $T_{(0369)}$, will continue to influence musical developments. The latter path is the one we will follow.

[22] Although the inversional transformational model underlies the pitch realization of both hexachordal streams in section four, besides the key confirming pitch, A♯3, another pitch, A5, deviates from its predetermined place in register. Of course, pitch-class 9 is represented by the repeated A7, the highest note in the passage, but A5 does not put in an appearance (see **Example 15**). However, the contour of the topmost line of the center hexachordal stream, especially the contour of the section's last four notes certainly suggests that A5 could be this line's goal. In fact, the A5 goal is achieved as the highest note of first chord following the double bar. The new chord is a member of set class 4-20[0158]. It begins the B section of the work, it holds the key to how both $T_{(0369)}$ progressions continue, and it sets the stage for future developments.

[23] Before the hexachordal progression through the T_1 $T_{(0369)}$ matrix was interrupted by the B 6-32[024579] hexachord, the expected hexachord was B♭ 6-32[024579]. Since nearly all the hexachords of the center stream are expanded to the member of 7-35[013568t] implied by the key signature, the {B♭, D, F, A} [0158] tetrachord can easily substitute for the missing B♭ 6-32[024579]/B♭ 7-35[013568t] complex by means of inclusion relationships. This chord also shares a deep voice leading connection with the four previous sections. At one level in fact, it is the culmination of that voice-leading pattern. As **Example 16** illustrates, placing the four inversional transformational models underlying sub-sections one through four of section A side by side and in the order of their appearance reveals that the highest notes of each model (beamed together in the example) nearly form the {B♭, D, F, A} [0158] tetrachord. Simply moving the B4 of the D♭ model a half step lower, a voice leading motion that has knitted sections together, transforms the beamed notes of the models into the [0158] tetrachord.

[24] Although inclusion relations and voice leading connections make a compelling case for associating the {B♭, D, F, A} [0158] tetrachord and the B♭ 6-32[024579] hexachord, the B♭ 4-20[0158] tetrachord and its companions at the beginning of the B section share an even more important syntactic connection with the TC model underlying sections one through four that solidifies their connection. The same T0369 cycle that generates an aggregate from a 3-6[024] trichord also generated an aggregate from a subset of the 4-20[0158] tetrachord, the 3-4[015] trichord (see **Example 17a**). Extending the process to the 4-20[0158] tetrachord simply produces a repetition of one of the (0369) generating cycles (see **Example 17b**). (At this point, some readers may wish to examine and compare Examples 32 and 34 (below), because they more explicitly illustrate the transformational relationship between object and process that link each of the matrices.) Although aggregates with repeated pitches are often called weighted aggregates in the literature, it is not a concept that applies to the present context. Once again, aggregates, in the present context, are compositional spaces that are not dependent on the mapping of one pitch for each pitch class.

[25] It should now be obvious that the other 4-20[0158] tetrachords accompanying the B♭ 4-20[0158] tetrachord at the beginning of the B section are the remaining members of the T_0 $T_{(0369)}$ [0158] matrix. The other three tetrachords all share the same relationship with a member of the T_1 $T_{(0369)}$ model of hexachordal progression that the B♭ 4-20[0158] tetrachord shares with B♭ 6-32[024579]. Therefore, if each [0158] tetrachord is a substitute for a hexachord in the T_1 $T_{(0369)}$ model of hexachordal progression, and if each tetrachord is functionally equivalent to the hexachords they substitute for, then the complex of 4-20[0158] tetrachords that begins the B section of the work both completes the hexachordal progression of the center stream outlined in Example 12 and recaps it in a near retrograde.

[26] Example 12 demonstrated that by collapsing each hexachord of the center stream to its first pitch class, the cycle generating the hexachords, (147t), also represents the progression of the hexachords. The same process applied to the pitch realizations of the 4-20[0158] tetrachords produces a reordering of the same cycle, <t471>, which is a near retrograde of the generating cycle for the hexachords. Although collapsing the pitch realizations of the 4-20[0158] tetrachords as "major seventh chords" into their "roots" produces a generating cycle of (147t), the cycle generating the pitch-class counterparts of 4-20[0158] tetrachords in Example 17b is (0369). Consequently, it always the second column of pitch-class matrices, such as the second column of Example 17b, that links a pitch-class complex of 4-20[0158] tetrachords to their hexachordal counterparts. Applying the transformations T_1 and T_2 to the original 4-20[0158] tetrachordal matrix generates two more matrices, which when combined with the T_0 matrix, produce all twelve transpositions of the 4-20[0158] set class. (See **Example 17c** and again, the interested reader may also wish to see Example 34 (below), which illustrates the transformational relationships more explicitly.) The hexachordal counterpart of the T_1 $T_{(147t)}$ tetrachordal matrix is the $T_{(258e)}$ hexachordal matrix, and the hexachordal counterpart of the $T_{(258e)}$ tetrachordal matrix is the $T_{(0369)}$ hexachordal matrix.

[27] The immediate appearance of hexachords B♭ 6-32[024579] and E 6-32[024579] in quick succession following the 4-20[0158] tetrachordal complex solidifies the relationship between the tetrachords and their hexachordal counterparts, since

these hexachords are the counterparts of the first two 4-2[0158] tetrachords, B♭ and E (see **Example 18**). The return of the hexachordal material initiates a new compressed phase of hexachordal development. The pitch realization of the hexachordal pair recalls the developments and structures outlined in sections one through four, while the compressed time presentation sets those relationships into relief. The alternation of 4-20[0158] complexes with hexachordal pairs that characterizes the continuation of the "development section" continues developing associations between the complexes, and it develops the implications of the interloping B 6-32[024579] from the end of the A section. As the square boxes labeled 4-20[0158] in **Example 19** illustrate, the 4-20[0158] tetrachords progress through the T_0, T_1, and T_2 $T_{(0369)}$ [0158] matrices thereby presenting all twelve transpositions of the set class. This progression introduces another level of aggregate partitioning. Here all the transpositions of a set are partitioned into groups that share an identical generative process and are related to each by transposition as well.

[28] The 6-32[024579] hexachords begin a similar progression and process in between the 4-20[0158] complexes. In this section, the hyperaggregate of transformations is not completed, but the implication that the interloping B 6-32[024579] hexachord's cycle will continue at some point is fulfilled (see Example 19). This is the only hexachordal cycle to be completed, in fact. A nice compositional detail connecting the end of the A section with the T_2 6-32[024579] matrix is the progression through the matrix's hexachords. The pitch realization of the final B 6-32[024579] hexachord is identical to its counterpart at the end of section A (see Example 19). Although all the hexachords of the T_2 6-32[024579] matrix occur in the progression completing the cycle, the cycle is still incomplete in another respect. The hexachords of the T_2 6-32[024579] occupy two different registral streams suggesting completion of the streams will occur at a later point in the music. The next hexachord to appear in the upper stream would be D 6-32[024579]. The D 6-35[02468t] whole tone hexachord that follows the final incomplete 4-20[0158] complex substitutes for the outer stream D 6-32[024579]. Essentially, D 6-35[02468t] functions as a transitional collection leading to the section that completes the 6-32[024579] hyperaggregate.

[29] The process of interpolation that began at the end of section A and characterized the beginning of the developmental section B continues as the 6-32[024579] hexachords complete their hyperaggregate (see **Example 20**). The T_1 matrix of 6-32[024579] hexachords, which was left incomplete in section A begins the progression through the hyperaggregate (**Example 21**). It is significant that the progression begins with E 6-32[024579] and moves directly to B♭ 6-32[024579] completing the progression in a direct way that was left incomplete at the end of the A section by the interpolated B 6-32[024579] hexachord. The hyperaggregate is completed by the appearance of the T_0 and T_2 6-32[024579] hexachordal matrices. The rearrangement of the transformational profile of the hexachordal complexes underscores the (0369) generative mechanism that underlies both the 6-32[024579] hexachordal and 4-20[0158] tetrachordal complexes. In section A, the hexachords within a stream moved through the ic 3 cycle while T_6 links the separate streams. Hexachords within the right and left hand parts of each complex still move along the ic 3 cycle and T_6 still links right and left hand streams, but the ordered succession of hexachords created by the interaction of streams follows the transformational path T_6—T_3—T_6 or (3906), for example. Each of the 4-20[0158] tetrachordal complexes in Example 19 follows the same transformational path.

[30] The new 3-4[015] interloper that separates the first two hexachordal complexes is, of course, related to the 4-20[0158] complexes by inclusion, but its new association with ic 6 foreshadows the translation of an earlier generative schema into the pitch dimension. After the final 6-32 hexachordal complex completes the hyperaggregate, the interpolated material moves to the foreground expanding the 3-4[015] trichord and the 4-8[0156] tetrachord into 6-7[012678] hexachords (see **Example 22**). The first 6-7[012678] hexachord in the right hand part leads to a T_6 re-mapping of itself that is immediately followed its complement at T_9. The 6-7[012678] hexachords travel along the same T_6—T_3—T_6 or (3906) transformational path taken by the 4-20[0158] tetrachords and 6-32[024579] hexachords. The left-hand parts are also T_6—T_3—T_6 or (3906) related and complements of the right hand parts. The $T_{(0167)}$ generative schema underlying the 3-4[024] trichords of the IS model from section A translated into a member of set class 4-9[0167] is, of course, included in set class 6-7[012678].

[31] Set class 4-9[0167] emerges from the 6-7[012678] hexachordal cloud to become a substantial entity in the following section where the super set generated by T_6 related 4-20[0158] tetrachords is regenerated by means of Cohn's concept of modes of generability. Each 4-20[0158] tetrachordal complex at the opening of section B consist of two pairs of T_6 related 4-20[0158] tetrachords forming a member of set class 8-9[01236789], a member of the TINV set classes (see **Example 23**). As well as generating all the transpositions of set class 4-20[0158], the tetrachordal complexes generate all the transpositions of set class 8-9[01236789]. In the section following the 6-7[012678] hexachords (see **Example 24a**), two different tetrachordal set classes, 4-9[0167] and 4-23[0257], generate the same collection of 8-9[01236789] octachords that is essentially a rotated and retrograded version of the collection of 8-9[01236789] octachords in Example 23 (see **Example 24b**).

[32] Although 4-9[0167] emerges as a pitch class event in the developmental B section, it is the coda that makes explicit its dual nature and its generative connection to the 3-4[024] trichords and the 6-32[024579] hexachords. After another round of development progressing through the T_0, T_1, and T_2 $T_{(0369)}$ 6-32[024579] hexachordal matrices, G 6-32[024579], the T_6 complement of D♭ 6-32[024579], leads to the return of the A section (see **Example 25**). In spite of its surface differences produced by the rhythmic activation of the upper trichord of the 6-32[024579] hexachords, the return of the A section is structurally identical to its counterpart at the syntactic level. That is, it progresses through the same four IS structures in the same order as its counterpart, and the progression leads to the same complex of 4-20[0158] tetrachords that marked the beginning of the development. The change of function to coda is perhaps first indicated by the D♭ member of 4-20[0158] that begins the progression through the complex paired with its T_6 partner (see **Example 26**). As was demonstrated earlier, the hexachordal counterparts of each tetrachord is D♭ and G 6-32[024579], respectively, the two hexachords that immediately follow the first 4-20[0158] complex of the coda.

[33] The progression of 6-32[024579] hexachords in the coda corresponding to the progression of hexachords in the development that completed the hyperaggregate reveals the dual nature of 4-9[0167] and its generative connection to the 3-4[024] trichords and the 6-32[024579] hexachords. The succession of hexachords abandons the (0369) transformational path to pursue the work's other generative path, (0167) (see **Example 27**). (The interested reader may also wish to compare Example 27 with Example 31 (below), which illustrates how the same transformational path and the same trichordal objects generate different hexachordal objects by changing the transformational relationships relating hexachordal objects.) As if it is trying to bring a subconscious thought into consciousness, the final reference to the 8-9[01236789] octachord begins with a solo statement of 4-9[0167] (see **Example 28**). The work closes with the T_1 and T_2 6-32[024579] hexachordal matrices following intertwined (0369) transformational paths (see **Examples 29a and b**). While the T_1 path leads to and concludes on the D♭ 6-32[024579] hexachord that began the work, the T_2 path takes an unexpected but logical turn. As if trying to bring another subconscious process of the work into consciousness, the final hexachord of the T_2 path, B 6-32[024579], becomes the whole tone collection {B, A, G, F, E♭, C♯, B}. Of course, this is the whole tone collection forming the upper half of the IS model from Example 5b that underlies the work's opening.

[34] The appearance of 6-35[02468t] as the goal of the of the intertwined T_1 and T_2 6-32 [024579] hexachordal matrices following the (0369) transformational path gives rise to another view of the work's final progression. Rather than viewing the succession of 6-32 hexachords as following intertwined (0369) transformational paths, we can also view the progression as a sequence of incomplete 7-35 septachords whose collection defining pitch class is the bass note of the lower 3-6[024] trichord.[(11)] The succession of bass notes, A3, G3, E♭3, and D♭3, summarizes the new transformational path that relates the collections to each other and to the final 6-35 pitch-class set. In this new view, each pair of incomplete septachords (A-G and E♭-D♭) is related by T_2, and each pair of T_2 related septachords is related by T_6. This transformational path connects the progression of incomplete septachords to the final 6-35 collection, because three T_6 related pairs of pitch classes related to each other by T_2 generates the 6-35 collection. It should also be noted that the collection defining pitch classes (A, G, E♭, and D♭) of the final progression of incomplete 7-35 septachords are a subset of the final B 6-35 collection. The transformational path that concludes the work sums up many of the transformational processes at work in *Processional.*

[35] Examples 30 through 35 and the discussion that follows summarize, in the abstract, the transformational networks governing the exchange of object and process in *Processional.* Reinterpreting the pitch-class matrices as transformational networks reveals that the network of tritones generating the whole-tone collection and the replication of transformations from one nodal level at higher or lower levels is a feature shared by all the matrices and is a source of the work's coherence. Each of the T_6 nodes in a column formed by the nodes of nodes in **Example 30** generates 3-6[024] trichords whose union produces 6-35[02468t]. Applying T_1 to Example 30 produces the $T_{(0167)}$ matrix of 3-6[024] trichords (see **Example 31**), and applying example 30's supernode transformations to the new supernodes produces the other matrices generating all twelve transpositions of 3-6[024]. Simply changing the second level T_1 transformation to T_3 produces all the $T_{(0369)}$ matrices of 3-6[024] trichords (see **Example 32**). **Example 33** illustrates that applying T_5 to third level node of example 32 generates the hexachordal matrix, and applying T_2 twice to this new supernode generates all the hexachordal matrices. **Example 34** illustrates the 4-20[0158] matrices keep the second level transformation constant and change the third level transformations. Finally, **Example 35** illustrates how successive T_5 transformations of the T_6 node produce two T_6 related 4-23[0257] tetrachords whose union generates 8-9[01236789].

[36] The numerous generative transformational paths leading from
subset to superset is, as Cohn has noted, one of the most interesting features of the TINV family of sets. It should not be

surprising that the aggregate or 12-1 shares this property, since it is a member of the TINV family. Translating TINV sets into transformational networks that partition the aggregate, however, means the TC property of 12-1 is non-trivial. The same cannot be said for a non-partitioned aggregate. As well as bonding together members of TINV with sets from outside the collection, TC aggregate partitioning can also be a bridge across different means of generating larger sets from smaller ones. For example, the non-intersection of generative components that is a hallmark of TC aggregate partitioning is one of the defining features of Robert Morris' complement union property or CUP. In future work, I hope to demonstrate the general properties relating TC TINV sets as a subset of generalized CUP relations. That is, we can view the two methods of generating larger sets from smaller ones as concentric circles with the smaller world of TINV contained within the larger CUP world with CUP perhaps in a more general set of relations, such as K-nets.

[37] For example, in his generalization of CUP relations, Morris allows CUP to expand in two directions by relaxing the constraint that the intersection of the generating sets must be the null set and union of the members of the generating sets classes must produce a single set class.[12] The number of set classes generated by the generalized CUP relation is indicated by a superscript added to CUP. CUP^4, for instance signifies that the members of the generating set classes produce four different set classes when the intersection of the generating sets is the null set. *Processional*'s generative 3-6[024] trichord produces a CUP^4 relation, when the generating sets are both members of set class 3-6. The four hexachords produced by this CUP relation are 6-1[012345], 6-35[02468t], 6-32[024579] and 6-8[023457]. Two of the resultant hexachords, 6-1 and 6-35, are members of TINV, and the generation of these hexachords from the smaller TINV collection 3-6 is a function of Cohn's modes of generability. The latter two hexachords, 6-32 and 6-8, are not members of TINV, however, but the similar generative path from smaller to larger set demonstrates how the smaller collection of TINV sets connects to the larger world of sets that are not members of TINV. It also demonstrates another reason why, in the specific case at hand, the diatonic makes a good playmate for TINV sets. The CUP^4 relation also demonstrates how the 7-23[0234579] from the work's opening fits into the larger framework of the piece, since the CUP4 hexachord 6-8[023457] is a subset of the 7-23 hexachord. As was stated earlier, the 6-33[023579] hexachord from the work's opening is also a subset of 7-23, and the mutual inclusion of 6-8, 6-33, and 6-32 in 7-23 creates an indirect role for 6-8 as a unifying force in *Processional*. K-nets, however, reveal that 6-33 hexachord is one path through which the 3-6[024] trichord creates connections with other trichords.

[38] The pitch-class exchange of B2 for G♭3 in Example 6 produced a member of the hexachordal set class, 6-33 [023579]. As was the case with the opening 6-32[024579] hexachord, the pitch realization of the hexachord is as two trichords. In the treble clef, the pitch classes {D♭, E♭, F} are a member of 3-6[024], and the pitch classes in the bass clef(s) {B, A♭, B♭} are a member of set class 3-2 [013]. However, unlike the 6-32[024579], the two trichords forming 6-33[024579] are members of different set classes. The $T_{(0369)}$ transformational path through its aggregate generating power created a powerful bond between members of different set classes, such as the 3-6[024] and 3-4[015] trichords. Unfortunately, neither the $T_{(0369)}$ nor the $T_{(0167)}$ transformational paths can similarly unite members of set classes 3-6[024] and 3-2[013]. Although the trichords do not forge a connection under the work's more prominent transformational networks, they are members of a k-net that does have a significant link with the T_2 relationship that figures so prominently in *Processional*. **Example 36** demonstrates that the k-nets for each trichord are isographic. Specifically, they are a positive isography under the group automorphism <1,1>.[13] The positive isography is the result of dyad-class 2's inclusion in each trichord. Of course, the transformational interpretation of the dyad becomes the operation T_2. These network interpretations of the pitch-class sets demonstrate that the T_2 relationship that generates the 3-6[024] trichord and so many other sets in the composition can also cast a wider net over many of the work's more local events and relate them to the underlying processes governing the global procession. Of course, the other prominent transpositional relationships generating transformation structures in *Processional* will produce similar k-nets relating structures that contain dyads other than ic 2.

[39] Expanding the analytical field of view to include the interconnection of object and process often reveals the camouflaged bridges connecting concentric circles. In this new worldview, compositional design would determine the utility of a circle's structural properties, and it would determine movement between circles. Since there are bridges connecting circles, the circles themselves do not have to become equivalence classes. I hope my analysis *Processional* has demonstrated the importance of considering both objects and their interconnection with process in *Processional* and perhaps in Crumb's work as a whole. The interconnection reveals his compositional procedures extend beyond the boundaries of any particular circle, such as exclusive use of symmetrical set classes. The study of object and process in a Crumb work also reveals that aggregate-based atonal methods and composing with symmetrical referential collections are concepts that can peacefully coexist and reinforce each other. In the spirit of Hegel, *Processional* is a synthesis of compositional procedures that often assume the roles of thesis and antithesis in the dialectic. Of course, the same observations hold with regard to the superset levels within the TINV family, so a well-partitioned aggregate is just as good a referential collection as any of the other

supersets in the TINV family. That is, the properties that endow the whole tone and octatonic collections with special status within the TINV family are properties also possessed by a well-partitioned aggregate. Finally, I hope that I have shown the structural gap generating equivalence classes of composers may not be as large as it once appeared to be. Perhaps now it is just a similarity relation.

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Footnotes

1. Richard Bass, "Models of Octatonic and Whole-Tone Interaction: George Crumb and His Predecessors," *Journal of Music Theory* 38 (1994): 155–186.
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2. Bass, "Models of Octatonic and Whole-Tone Interaction," 186.
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3. Richard Cohn, "Properties and Generability of Transpositionally Invariant Sets," *Journal of Music Theory* 35 (1991): 1–32.
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4. Cohn, "Properties and Generability," 4.
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5. CUP is Robert D. Morris's term for the complement union property. He presented this work in the article "Pitch-Class Complementation and its Generalization," *Journal of Music Theory* 34 (1990): 175–245. K-net is just a shorthand term for a Klumpenhouwer Network. The general properties of k-nets are presented by David Lewin in the article "Klumpenhouwer Networks and Some Isographies that Involve Them," *Music Theory Spectrum* 12 (1990): 83–120.
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6. IC/B is Lewin's label free method of notating inversional operations. See David Lewin, "A Label-Free Development for Twelve-Pitch-Class Systems." *Journal of Music Theory* 21 (1977): 29–48.
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7. Hearing pitch events as occupying distinct registral steams affords each stream a degree of independence from the activities or processes unfolding in another stream. For example, pitches that are members of the same pitch class can have distinct functions determined by the registral stream they occupy. Therefore, the B2 and B4 of Example 6 are not merely octave duplications, they perform different functions determined by the process unfolding in their respective streams. This point will be reinforced in the above text and the text that follows.
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8. The significance of the 6-Z28 hexachord will be revealed later in the paper.
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9. This method of generating aggregates was introduced by Daniel Starr and Robert Morris in their two-part article "A General Theory of Combinatoriality and the Aggregate," *Perspectives of New Music* 16/1 (1977): 3–35; *Perspectives of New Music* 16/2 (1978): 50–84.
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10. As indicated earlier, a third stream unfolds in the outer registers of the piano that is initiated by the pitch B2.
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11. The precedent for interpreting hexachords as incomplete members of the set class 7-35 was, of course, established at the

work's opening. Hearing the hexachords within a septachordal context allows us to hear the A♭ 6-32 collection as invoking the D♭ 7-35 collection, which in turn allows us to relate the A♭ 6-32 collection to the work's opening hexachord. All of these relationship are reinforced by the immediate appearance of D♭ 6-32 following A♭ 6-32 (see **Example 29b**). The union of these two collections, of course, generates the implied D♭ 7-35 collection.
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12. Robert Morris, "Pitch-Class Complementation and its Generalization," 191–95.
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13. For a full explanation of the general properties of k-net isographies and the rules for generating network isomorphisms see Lewin, "Klumpenhouwer Networks and Some Isographies that Involve Them."
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