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Exploring Tetrachordal Voice-Leading Spaces Within and Around the MORRIS Constellation

Mark Sallmen

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ABSTRACT: Building on the work of Stephen Soderberg, Julian Hook, Robert Morris, and others, this article explores a wide variety of voice-leading transformations involving set types 4-27[0258], 4-18[0147], 4-13[0136], and 4-12[0236]. It considers tetrachordal connections between any two members of the same set class and all twenty-four ways to voice-lead each tetrachordal connection. The paper organizes these many possibilities and suggests compositional applications. It shows various ways to maintain control over the content of individual voices by constructing voice-leading spaces that involve a limited number of voice-leading transformations and rules for concatenating the transformations.

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[1] Building on Robert Morris's (1990) research on hexachordal ZC-relations, Stephen Soderberg (1998) identifies a constellation of ten hexachords that embed either one diminished seventh chord or two diminished triads. Soderberg divides the constellation, called MORRIS (or T-HEX), into four overlapping eight-hexachord sub-constellations based on tetrachordal subset content. The first of these sub-constellations, TRISTAN, includes the hexachords that embed two instances of set class 4-27[0258], the set class of the major-minor and half-diminished seventh chords. Similarly, constellations ZAUBER, AGITATION, and BROODING include the hexachords that embed two instances of set classes 4-18[0147], 4-13[0136], and 4-12[0236], respectively. Soderberg characterizes each of these tetrachordal set types as a "warp" of the diminished seventh chord. When the "warp index," *w*, is 1, the result is set class 4-27—that is, moving any pc of a diminished seventh chord by interval class (ic) 1 creates a member of set class 4-27. Similarly, setting *w* = 2, 4, and 5 creates set classes 4-18, 4-13, and 4-12, respectively. The article goes on to point out a general property of voice leading: in each hexachord the pair of tetrachords can be connected by holding two pitch classes in common and by moving two others by ±w. The cases involving 4-27 and ic 1 voice leading (TRISTAN) are familiar—ii$^{ø}{}^{4}_{3}$–V^{7}, the Tristan chord with resolution, A♯^{ø7}–B♭^{7} at the beginning of Debussy's *Faune*, and others—and have been addressed in the theoretic literature by several authors.(1) **Example 1** presents the MORRIS constellation and its four overlapping sub-constellations, henceforth called

$MORRIS_1$, $MORRIS_2$, $MORRIS_4$, and $MORRIS_5$, with each subscript indicating the warp index, *w*.

[2] Taking Soderberg's MORRIS constellation as a starting point, this study explores a wide variety of voice-leading transformations involving set types 4-27, 4-18, 4-13, and 4-12. It starts with the tetrachordal voice-leading transformations that produce the $MORRIS_w$ hexachords—the nine ways that a tetrachord may be connected to another in the same set class by holding two pitch classes in common and by moving two others by $\pm w$. (For each value of *w* there are nine transformations but only eight hexachord types because two of the transformations yield the same hexachord type.) The article then greatly expands the scope of inquiry, not only by allowing a given tetrachord to connect to *any* member of the same set class, but also by considering all twenty-four ways to voice-lead each of these twenty-four tetrachordal connections. As a result, each of these much larger voice-leading spaces—$MORRIS+_1$, $MORRIS+_2$, $MORRIS+_4$, and $MORRIS+_5$ —contains 576 (= 24 × 24) voice-leading transformations. Each $MORRIS_w$ is a subset of its corresponding $MORRIS+_w$.

[3] The paper provides an organized view of the entire $MORRIS_w$/$MORRIS+_w$ system, but it also develops other voice-leading spaces within $MORRIS_w$/$MORRIS+_w$, each of which involves only a few of the $MORRIS_w$/$MORRIS+_w$ voice-leading transformations. The additional voice-leading spaces usually also include rules for requiring, preferring, allowing, or forbidding one transformation to follow another. Although the number of potential voice-leading spaces is infinite,(2) my reasons for creating the ones I do are simple—to exercise melodic control over intervallic characteristics of the voices and to exercise harmonic control, not only of the individual tetrachords, but over the total pc content of sets of adjacent tetrachords. For example, in one space defined below a set of six $MORRIS_5$ voice-leading transformations can be concatenated in any order but remain within a single octatonic collection, in another space four $MORRIS+_4$ transformations are strung together so that one voice descends chromatically while the other voices also move stepwise, and in a third example ten $MORRIS+_2$ transformations are arranged so that each voice is saturated with a different interval class.

[4] For an introduction to the variety exhibited by the voice-leading transformations within the $MORRIS+_w$ universe, consider the musical excerpts in Example 2. **Example 2a** provides a transformation from $MORRIS_2$, a pair of 4-18 tetrachords connected by ic-2 voice leading in two voices and by common tones in the other two, producing 6-30[013679].(3) An example from $MORRIS+_1$, the excerpt from Debussy's *Faune* in **Example 2b** includes a pair of 4-27 tetrachords (G♯$^6_5$–B^{ø7}) with the same ic in each voice (ic 1) and an overall octatonic collection.(4) The excerpt from Berg's *Wozzeck* in **Example 2c** provides a case from $MORRIS+_2$, in which a pair of 4-18 tetrachords state ic 2 in two voices and ic 4 in the other two. **Example 2d** identifies a voice-leading transformation from $MORRIS+_4$, a pair of 4-13 tetrachords with a different ic in each voice (ics 3, 0, 5 and 2). As shown in **Example 2e**, this four-voice model appears within the opening phrase of the vocal line in the third song of Berg's *Altenberg Lieder*. Despite the linear realization, individual voices of the four-voice model are clear because each 4-13 articulates the same pitch contour 2–1–0–3. For example, ic 3 is clear in the "upper voice" because it is articulated by B♭4, the last and highest pitch of the first 4-13, and C♯5, the last and highest pitch of the second 4-13.

[5] The article builds on previous research in transformational theory and voice leading in several ways. First, following work of Julian Hook and others that deals with triads and seventh chords, the paper uses contextually-defined schritt (S_n) and wechsel (W_n) transformations to define mappings between tetrachords within the same set class.(5) Second, in his article on voice leading with set type 4-27, Adrian Childs (1998) develops transformation labels that identify contrary/parallel motion in the moving voices as well as the harmonic interval classes formed by the sustained and moving voice pairs; I employ an adapted version of this labeling system to address all four tetrachord types. Third, branching out from the research on parsimonious voice leading—which focuses on stepwise melodic motion and tends to emphasize situations where moving voices articulate the same melodic interval class—this paper embraces both stepwise and non-stepwise voice leading and includes cases where voices move by the same or by different interval classes.(6)

[6] Further, building on Robert Morris's research in voice leading and compositional spaces, the paper utilizes T-matrices to investigate myriad voice-leading possibilities; develops several ways to measure voice leading, one of which is an unordered count of voice-leading interval classes that recalls Morris's "voice-leading lists"; and suggests compositional applications of the theory.(7) All voice leading in the article is one-to-one and onto, that is, within each tetrachord each pc appears in exactly

one voice and pcs are neither doubled nor omitted. (8) Finally, as is customary throughout much of the literature, networks graphically depict abstract and literal voice-leading spaces. (9)

[7] The paper is in five parts. Part I offers preliminary definitions of the transformation and voice-leading concepts used throughout the remainder of the paper. Part II introduces the MORRIS$_w$ voice-leading spaces. Part III suggests how to concatenate the voice-leading transformations within each MORRIS$_w$ while being sensitive to recurring patterns of moving and holding within individual voices, aggregate completion, and related issues. Part IV introduces MORRIS+$_1$, MORRIS+$_2$, MORRIS+$_4$, and MORRIS+$_5$, categorizes their voice-leading transformations according to various properties so that a coherent view of the entire system emerges, suggests several compositional applications, and provides an analysis of an excerpt from Bartok's String Quartet No. 6. Part V shows how to put together lengthy strings of MORRIS+$_w$ voice-leading transformations while maintaining control over the intervallic content of individual voices.

Part I. Preliminary Definitions

[8] Part I defines several features of pitch-class sets (prime and inverted orientations, conventional ordering, set members) to prepare for a discussion of schritt and wechsel transformations (S_n/W_n). Three types of intervals are then defined; two are familiar (pc interval, interval class) and one is new (contextual interval). This sets the stage for the definition of voice-leading transformations (VL), as well as for ordered and unordered lists of interval classes and contextual intervals. Part I concludes with discussions of T-matrices and inverse- and Z-related VL.

[9] A set is said to be in *prime orientation* if it is related to its prime form by T_n and in inverted orientation if related to its prime form by T_nI. For example, within set class 4-18[0147], {1, 4, 7, 0}, {2, 5, 8, 1}, and {3, 6, 9, 2} are in prime orientation because they are related by T_n to their set class prime form and {7, 4, 1, 8}, {6, 3, 0, 7}, and {5, 2, B, 6} are in inverted orientation because they relate to the prime form via T_nI. We will adopt the conventional ordering of pcs within tetrachordal sets instanced by the sets just mentioned. That is, for sets in prime orientation the first three pcs articulate an ascending root-position diminished triad (e.g. {1, 4, 7, 0}), and for sets in inverted orientation the first three pcs articulate a descending root-position diminished triad (e.g. {7, 4, 1, 8}). Based on this conventional ordering, individual pcs within a set are designated as set members (sm) 1, 2, 3, and 4, respectively. That is, in {1, 4, 7, 0}, pc 1 is set member (sm) 1, pc 4 is sm 2, pc 7 is sm 3, and pc 0 is sm 4. Within {7, 4, 1, 8}, pc 7 is sm 1, pc 4 is sm 2, pc 1 is sm 3, and pc 8 is sm 4. (10)

[10] A *schritt* pcset transformation, S_n, is defined to articulate T_n when applied to a set in prime orientation, but T_{-n} when applied to a set in inverted orientation. For example, S_1 transforms {1, 4, 7, 0} into {2, 5, 8, 1}, which articulates T_1, and S_1 transforms {7, 4, 1, 8} into {6, 3, 0, 7}, which articulates T_B. S_0, the identity transformation, transforms any set onto itself. Concerning *wechsel* transformations, when W_n transforms a set in prime orientation into an inverted one the embedded diminished triads articulate T_n, but when it transforms an inverted set into a prime one the embedded diminished triads articulate T_{-n}. For example, W_1 transforms {0, 3, 6, 1} into {7, 4, 1, 6}, within which {0, 3, 6} and {1, 4, 7} articulate T_1; W_1 transforms {6, 3, 0, 5} into {B, 2, 5, 0}, in which {0, 3, 6} and {B, 2, 5} articulate T_B. It would have been possible to define the W subscripts by chronicling the movement of any referential pc within the tetrachords. The use of the diminished triad strikes me as best because it allows a single rule for all four set types and has other advantages. For instance, it engages Straus's 1997 notion of "near-transposition" ($*T_n$); that is, each W_n articulates $*T_n$ when applied to a prime set and $*T_{-n}$ when applied to an inverted set because it moves all but one of the set's pcs by the same pc interval. (11)

[11] In order to compare features of the different MORRIS+$_w$ voice-leading spaces, this paper often characterizes a set of transformations in terms of *w*, the warp index. For example, transformation S_w refers to MORRIS+$_1$ S_1, MORRIS+$_2$ S_2, MORRIS+$_4$ S_4, and MORRIS+$_5$ S_5. Similarly, W_{w+3} refers to MORRIS+$_1$ W_4, MORRIS+$_2$ W_5, MORRIS+$_4$ W_7, and MORRIS+$_5$ W_8.

[12] This paper uses three types of voice-leading intervals. The first two types are familiar: given a voice in which pc x in tetrachordal set X is followed by pc y in tetrachordal set Y, the *pc interval* from pc x to pc y is y−x mod 12 and the *interval class* (ic) between pcs x and y is |y−x| mod 12. (Henceforth all arithmetic is considered to be mod 12 unless otherwise noted.) For example, the pc interval from 2 to 5 is 3, from 7 to 4 is 9, and from 1 to B is A, and the interval class between 2 and 5 is

3, between 7 and 4 is 3, and between 1 and B is 2. Like the subscripts of the S_n and W_n transformations, the third type of interval depends on the (prime or inverted) orientation of the initial set. If set X is in prime orientation the *contextual interval* (ci) from x to y is y−x (the same as the pc interval), but if X is in inverted orientation then the contextual interval from x to y is x−y (the inverse of the pc interval). For example, if pc 2 occurs within a set in prime orientation, such as {0, 3, 6, 2}, then the ci from pc 2 to pc 5 is 3; but if pc 2 appears within a set in inverted orientation, such as {2, B, 8, 0}, then the ci from pc 2 to pc 5 is 9. (The orientation of set Y does not affect the ci.)

[13] Given any pair of tetrachordal sets X and Y, there are twenty-four ways to organize pcs into four voices so that each pc in each set appears precisely once. Therefore, there are twenty-four ways to voice-lead each S_n/W_n transformation. The paper identifies each of these possibilities using *ordered lists of contextual intervals* (*oci*). For example, oci 8259 indicates that sm 1 of X moves by ci 8, that sm 2 of X moves by ci 2, that sm 3 of X moves by ci 5, and that sm 4 of X moves by ci 9. A *voice-leading transformation*, hereafter VL, can be minimally defined by tetrachord type and oci. (12) For example, 4-13[0136] and oci 8259 define a VL that can be articulated by the set of voices {0–8, 3–5, 6–B, 1–A}. That is, sm 1 of X is pc 0, which moves by ci 8 to pc 8, sm 2 of X is 3, which moves by ci 2 to pc 5, sm 3 of X is 6, which moves by ci 5 to pc B, and sm 4 of X is 1, which moves by ci 9 to pc A. Throughout the paper, we will indicate tetrachord type with a voice-leading space (such as MORRIS+_4 for 4-13) and include the pcset transformation that the VL articulates, creating VL labels such as MORRIS+_4 W_5 8259. (13) **Example 3a** provides three realizations of this VL. The first two use the set of voices {0–8, 3–5, 6–B, 1–A} and the third {5–9, 2–0, B–6, 4–7}, which begins with a chord in inverted orientation. (14)

[14] As various VL are compared and contrasted, it is useful to provide other ways to characterize them. The *unordered list of contextual intervals* (*uci*) uses superscripts to count the number of times each ci appears within a VL; for instance MORRIS+_4 W_5 8259 articulates uci $2^1 5^1 8^1 9^1$. The *unordered list of interval classes* (*uic*) converts ci to ic; for example MORRIS+_4 W_5 8259 yields uic $2^1 3^1 4^1 5^1$. On a larger level, these uci and uic organize into types based on their superscripts. There are five uci types: x^4, x^3y^1, x^2y^2, $x^2y^1z^1$, $x^1y^1z^1q^1$, where x, y, z, and q are different ci. For example, uci $2^1 5^1 8^1 9^1$ (and all others with one instance of four different ci) are of the type $x^1y^1z^1q^1$. There are also five uic types: $|x|^4$, $|x|^3|y|^1$, $|x|^2|y|^2$, $|x|^2|y|^1|z|^1$, $|x|^1|y|^1|z|^1|q|^1$, where $|x|$, $|y|$, $|z|$, and $|q|$ are different ic. For instance, uic $2^1 3^1 4^1 5^1$ and all others with one instance each of four different ic are of the type $|x|^1|y|^1|z|^1|q|^1$. Concerning the relation of a VL's uci and uic, when all ci in a uci articulate different ics (e.g. uci type $x^1y^1z^1q^1$ where $x \neq -y$, $x \neq -z$, $x \neq -q$, $y \neq -z$, $y \neq -q$ and $z \neq -q$) the relation between uci type and uic type is straightforward, as in the case of uci $2^1 5^1 8^1 9^1$ and uic $2^1 3^1 4^1 5^1$. But when different ci articulate the same ic (e.g. $x = -q$), then the uci and uic superscripts will differ. For example, MORRIS+_4 W_5 824A articulates uci $2^1 4^1 8^1 A^1$ but uic $2^2 4^2$.

[15] A *T-matrix* provides an efficient way to consider all VL that articulate a given transformation. Set X appears to the left of the matrix in conventional ordering and set Y appears above the matrix in conventional ordering. Matrix rows are labeled 1–4 from top to bottom to correspond to the set members of X, and columns are labeled 1–4 from left to right to correspond to the sm of Y. Matrix position (j, k) contains the contextual interval from sm j of X to sm k of Y. A one-to-one and onto mapping of X onto Y involves four matrix positions that include each matrix row and column precisely once. As an example, consider the matrix for MORRIS+_4 W_5, where X = {0, 3, 6, 1} and Y = {B, 8, 5, A}, given as **Example 3b**. The boldface and underlined ci within the matrix point out VL MORRIS+_4 W_5 8259—ci 8, 2, 5, 9 at matrix positions (1, 2), (2, 3), (3, 1), and (4, 4), respectively. (15) Notice that the notation MORRIS+_4 W_5 8259 does not explicitly indicate the various sm of Y, but that viewing the VL in its matrix context does make them explicit—the sm of Y are the column numbers in the matrix positions. For example, ci 8 at matrix position (1, 2) means that sm 1 of X moves to *sm 2 of Y* via ci 8, ci 2 at matrix position (2, 3) means that sm 2 of X moves to *sm 3 of Y*, and so forth. Later in the paper, when the sm of Y is not vital to the discussion I use the compact notation (e.g. MORRIS+_4 W_5 8259), but when the goal sm is vital—as in Part V when it comes time to control the intervallic content of a single voice in a lengthy chain of VL—I list ci along with their matrix position (e.g. ci 8 @ (1, 2), etc.). **Example 3c** lists the twenty-four VL that articulate transformation MORRIS+_4 W_5, along with their uci and uic interpretations.

[16] Two VL are *inverses* if and only if applying one then the other results in an overall S_0 0000. Inverse-related wechsel VL articulate the same pcset transformation and the same uci, whereas inverse-related schritt VL articulate inverse pcset

transformations and inverse sets of ci; for example, the inverse of MORRIS+$_4$ W_5 8259 is MORRIS+$_4$ W_5 5829 and the inverse of MORRIS+$_4$ S_1 1186 is S_B BB64. Some VL are their own inverses, as with MORRIS+$_4$ S_6 0606 and MORRIS+$_4$ W_4 8286 (**Example 3d**).[16] VL are said to be *Z-related* if and only if they articulate the same pcset transformation and uic. Inverse-related wechsel VL are Z-related by definition, but there are often non-trivial Z relations. For instance, MORRIS+$_4$ W_5's 87BA, A8B7, B247, and B724 each articulate uic $1^1 2^1 4^1 5^1$, two pairs of inverse VL that create a Z-quadruple (**Example 3e**).[17]

Part II. VL within the MORRIS$_w$ Voice-Leading Spaces

[17] Part II derives and lists the MORRIS$_w$ VL, then introduces the Childs-derived labels, which help to compare and contrast various VL. **Example 4** features four lattices that facilitate deriving the MORRIS$_w$ VL—the nine ways to transform each MORRIS$_w$ tetrachord into another member of the same set class by holding two pitch classes in common and by moving each of the other two pitch classes by interval class *w*. Each lattice has <*w*>-cycle rows and <3>-cycle columns. Within each lattice, each row represents a voice, so that voice leading by ci *w* is modeled by horizontal movement to the right to an adjacent pc within a row and voice leading by ci −*w* by movement to the left. Vertical pc moves are forbidden. Having one pc in each row creates a tetrachord, which is a member of the MORRIS$_w$ tetrachordal set class if and only if three pcs are in one column and one pc is in an adjacent column. Taking MORRIS$_5$ as an example, starting with {0, 3, 6, 2}, there are twenty-four possible ways to move two voices by ic 5—six possible pairs of moving voices and two possible directions for each moving voice—but only nine of these create another 4-12. Since pc 2 is in a column by itself it is helpful to organize these nine possibilities around what pc 2 does. First, if pc 2 leads to pc 7 a dead end is reached because pc 7 would be *two* columns away from the other pcs and with only one move remaining it is impossible to move all three remaining voices to an adjacent column. If pc 2 leads to the left, to pc 9, there are six other single moves that generate another 4-12. Three of these involve moves to the right (0–5, 3–8, and 6–B) and the other three involve moves to the left (0–7, 3–A, and 6–1). Third and finally, if pc 2 does not move, then two of the other voices need to be led to the right to generate another 4-12. There are three ways to do this (0–5 with 3–8, 0–5 with 6–B, and 3–8 with 6–B). Overall, there are three cases where both moving voices articulate ci 5 (move to the right) and so the uci is $0^2 5^2$, three cases where both moving voices articulate ci 7 (move to the left) and so the uci is $0^2 7^2$, and three cases where one moving voice articulates ci 5 and the other ci 7 resulting in uci $0^2 5^1 7^1$. In the same way one could generate the other three sets of MORRIS$_w$ transformations.

[18] **Example 5** illustrates all of the resulting MORRIS$_w$ VL in staff notation, along with their uci and uic interpretations. Each set of nine VL divide into three groups based on uci. In each case, S_3 *w*00(−*w*), S_6 0*w*0(−*w*), and S_9 00*w*(−*w*) articulate uci $0^2 w^1 (-w)^1$; W_3 (−*w*)00(−*w*), W_6 0(−*w*)0(−*w*), and W_9 00(−*w*)(−*w*) articulate uci $0^2 (-w)^2$; and W_{3+w} 0*ww*0, W_{6+w} *w*0*w*0, and W_{9+w} *ww*00 articulate uci $0^2 w^2$. In MORRIS$_5$ for example, S_3 5007, S_6 0507, and S_9 0057 articulate uci $0^2 5^1 7^1$; W_3 7007, W_6 0707, and W_9 0077 articulate uci $0^2 7^2$; and W_8 0550, W_B 5050, and W_2 5500 articulate uci $0^2 5^2$. These three groups of three correspond to the three general lattice-movement possibilities mentioned above: one move in each direction, two moves to the left, and two moves to the right, respectively. In each MORRIS$_w$, all nine transformations articulate uic $0^2 w^2$.[18]

[19] The example also lists the hexachord types that are the focus of Soderberg's article. Four set types appear in each MORRIS$_w$. S_3 *w*00(−*w*) and S_9 00*w*(−*w*) state 6-27[013469], S_6 0*w*0(−*w*) states 6-30[013679], and W_3 (−*w*)00(−*w*) and W_9 00(−*w*)(−*w*) state 6-Z42[012369] and 6-Z29[023679]. Two hexachords appear in only two constellations: W_6 0(−*w*)0(−*w*) generates 6-Z28[013569] in MORRIS$_1$ and MORRIS$_5$ and 6-Z45[023469] in MORRIS$_2$ and MORRIS$_4$.

[20] The example also provides Childs-derived labels, in which each digit indicates an interval class, "h" stands for held, "c" for contrary motion and "p" for parallel. For example, the label for MORRIS$_4$ S_3 4008 is h3c15, which indicates that the *held pcs* (3 and 6) articulate ic 3, the *moving voices* (0–4 and 1–9) progress in contrary motion, and create what I will call the *moving dyads* ({0, 1} and {4, 9}), which articulate ic 1 then ic 5. Such labels highlight five features of the VL. First, ics 3 and 6 play a salient role: each VL features ic 3 or 6, either as its held dyad or as its parallel moving dyads. Second, in MORRIS$_1$ and MORRIS$_5$, if a held dyad articulates an odd ic the moving dyads articulate even ics, and if the held dyad is even the moving dyads are odd (e.g. h3c24, h1p66, h6c11, h4p33), but in MORRIS$_2$ and MORRIS$_4$ the held and moving dyads within the

same VL are either all odd or all even (e.g. h3c15, h3p11, h6c22, h4p66). Third, within each MORRIS$_w$, even though S_6 $0w0(-w)$ and W_6 $0(-w)0(-w)$ are different VL—one involves contrary motion and the other parallel—they involve the same held ic and the same moving dyad ics, as with MORRIS$_5$'s S_6 0507 = h$\underline{6}$c$\underline{11}$ and W_6 0707 = h$\underline{6}$p$\underline{11}$.

[21] Fourth, each Childs-derived label that includes ic 3 appears in two MORRIS$_w$ systems; that is, VL that articulate S_3, S_9, W_3, W_9, W_{3+w}, and W_{9+w} in MORRIS$_w$ have the same set of held and moving dyads as the corresponding transformation in MORRIS$_{6-w}$. For example, MORRIS$_1$ S_3 100B and MORRIS$_5$ S_3 5007 both articulate h3c24; MORRIS$_1$ W_3 B00B and MORRIS$_5$ W_3 7007 both articulate h3p22; and MORRIS$_1$ W_4 0110 and MORRIS$_5$ W_8 0550—which are W_{3+w} $0ww0$ in both cases—articulate h2p33. Fifth and finally, W_3 and W_{3+w} VL swap their held and moving ics as do W_6 and W_{6+w}, and W_9 and W_{9+w}, creating what we will call *obverse VL pairs*.[19] For example, MORRIS$_2$ W_9 00AA = h3p55 and W_B 2200 = h5p33 are obverses. Any VL followed by its obverse results in an overall S_w $wwww$ or S_{-w} $(-w)(-w)(-w)(-w)$. MORRIS$_2$ W_B 2200 then W_9 00AA articulates an overall S_2 2222, while MORRIS$_2$ W_9 00AA then W_B 2200 an overall S_A AAAA. Three other obverse pairs are shown in **Example 6**.

Part III. Four Applications of MORRIS$_w$ VL

[22] There are innumerable ways to combine VL within a given MORRIS$_w$. As an introduction to this topic, part III explores four situations: it creates series of MORRIS$_1$ VL in which one voice is held throughout, concatenates MORRIS$_5$ VL in an octatonic context, and generates twelve-tone structures from MORRIS$_2$ and MORRIS$_4$ VL. Since long strings of VL will be considered and since each pcset transformation within the MORRIS$_w$ universe is associated with only one VL, I will employ a short hand notation where a pcset transformation label stands for its corresponding VL; for example MORRIS$_1$ W_4 stands for MORRIS$_1$ W_4 0110, MORRIS$_1$ S_3 stands for MORRIS$_1$ S_3 100B, and so forth.

[23] One way to limit the possibilities for VL concatenation is to control the motion of one or more particular voices. For example, it is possible to arrange VL so that a particular voice is always held. As an example, the network in **Example 7a** suggests how to assemble MORRIS$_1$ VL so that pc 2 is held throughout. Network nodes identify the eight members of set class 4-27 that include pc 2 and lines connecting the nodes show which chords may be connected by MORRIS$_1$ VL. Each line is bidirectional, so that, for instance, D^7 may lead to $D^{\varnothing 7}$ and vice versa. In most cases, both directions articulate the same transformation, as with W_4 which transforms D^7 into $D^{\varnothing 7}$ and vice versa; but in four cases, one direction articulates S_3 and the other its inverse, S_9 (e.g. S_3 transforms G^7 into E^7 and S_9 transforms E^7 into G^7). Since D^7 and $E^{\varnothing 7}$ each connect to three other chords, and the other six chords each connect to five others, there are still a huge number of possible paths through this network.

[24] To reduce the possibilities further we can stipulate that the path begins and ends at D^7 and includes each other chord precisely once, and that the voice beginning with pc 0 alternates holding and moving. Since each path begins with D^7, and by holding pc 0, the only other chord that includes pc 0, $D^{\varnothing 7}$, must be the second chord. Pc 0 must then move by ic 1. Since no chords on the network include C♯, the next chord must be one of the four that includes pc B ($B^{\varnothing 7}$, E^7, G^7, $G\sharp^{\varnothing 7}$). Pc B then holds, and so the next chord must also be from this group. Since each chord in this group connects to two others in the group there are always two possibilities. Pc B must then move by ic 1. It may not move back to pc 0 (because there are no new chords that include pc 0) so it must move to pc A, which is harmonized by the only two chords that include it $B\flat^7$–$E^{\varnothing 7}$ (or vice versa). In similar fashion the path returns to D^7 via the two chords that have not yet been included. All in all, there are ten possible paths, given in **Example 7b**. Two paths contain only wechsel VL and eight paths contain a mixture of wechsel and schritt VL. **Example 7c** realizes two of these paths in staff notation. In each case the held pc 2 appears in the bass voice and the pc-0 voice in the soprano.

[25] Another way to restrict choice is to limit the possible VL to fewer than nine; say, for example, the six that produce hexachords that are subsets of the octatonic collection. In MORRIS$_5$ for instance, these are S_3, S_6, S_9, W_8, W_B, and W_2. Any string involving these VL stays within the same octatonic collection. The polyrhythmic texture in **Example 8** provides one such series, W_8–S_9–W_8–S_3–S_3–W_2–S_9–W_2–S_9–W_8–W_2, which uses only four of the six octatonic VL but which includes all eight instances of 4-13 within one octatonic collection. In this and all such octatonic series each voice alternates pc intervals w and $-w$ and therefore oscillates between two pitch classes; this occurs because consecutive w (or consecutive

$-w$) would lead outside the octatonic collection.

[26] It is also possible to concatenate VL so that they create twelve-tone structures. For example, consider series of MORRIS$_2$ VL that embrace all twelve pcs as quickly as possible, that is, four pcs in the first chord and two new pcs in each of the next four. Put another way, each series uses each pc exactly once—a pc may be held from one chord to the next but once left it is not reiterated. Fulfilling these conditions is only possible when all four voices move in the same direction and so schritt VL, which involve contrary motion, are forbidden. [(20)] Since a series of wechsel VL creates an alternation between sets in prime orientation and those in inverted orientation, there must also be an alternation of contextual intervals 2 and A in order to keep the voices moving by the same pc interval. This means that if the first transformation is W$_3$, W$_6$, or W$_9$, which articulate uci 0^2A^2, then the next transformation must be W$_5$, W$_8$, or W$_B$, which articulate uci $0^2 2^2$. For example, transforming {0, 3, 6, B}, a set in prime orientation, by W$_3$ creates motion by pc interval A (0–A and B–9), which leads to {A, 3, 6, 9}, a set in inverted orientation. To continue motion by pc interval A, we need to choose W$_5$, W$_8$, or W$_B$; choosing W$_B$, for instance, creates moving voices 6–4 and 9–7, which lead to {A, 3, 4, 7}. At this point there are three possible continuations that complete the aggregate: W$_3$–W$_5$, W$_9$–W$_B$, and W$_6$–W$_8$.

[27] Musical realizations of these three complete series, W$_3$–W$_B$–W$_3$–W$_5$, W$_3$–W$_B$–W$_9$–W$_B$, and W$_3$–W$_B$–W$_6$–W$_8$, appear in the first system of **Example 9**. The second system presents the other four series that begin with W$_3$. There also exist seven series that begin with W$_9$ and six that begin with W$_6$. All twenty of these series feature the same set of four voices (0–A–8, 6–4–2, 3–1, and B–9–7–5) but with varying pc repetitions. For instance, in various series the 0–A–8 voice is articulated as 0–A–A–A–8, 0–A–A–8–8, 0–0–A–8–8, 0–0–A–A–8, and 0–0–0–A–8. Finally, all twenty of these series may also be played in retrograde, for a total of forty MORRIS$_2$ series that complete the aggregate as quickly as possible. [(21)]

[28] MORRIS$_4$ VL feature set type 4-13[0136] and voice leading by ic 4. Since 4-13 creates the pc aggregate when transposed by 4 and 8, these VL are ideally suited to produce twelve-tone designs. Consider the matrix in **Example 10a**, whose rows and columns are saturated with MORRIS$_4$ VL, each expressed as a series of three dyads. For example, {0, 1}–{3, 6}–{8, 9} in the top row expresses W$_3$ = h3p11. The held pcs, 3 and 6, appear in the middle dyad, surrounded by the moving dyads, so that both 4-13 are clear ({0, 3, 6, 1} and {3, 6, 9, 8}). The moving voices, 0–8 and 1–9, are articulated by pcs in the outer dyads. Completing the top row, {8, 9}–{B, 2}–{4, 5} and {4, 5}–{7, A}–{0, 1} replicate this W$_3$ transformation at T$_4$ and T$_8$, creating a twelve-tone cycle that wraps around to its starting point. Offset by one dyad, this row also thrice embeds W$_7$ = h1p33, the obverse of W$_3$ = h3p11: {3, 6}–{8, 9}–{B, 2} and its T$_4$ and T$_8$ transformations. The second-highest row is a retrograde rotated circle-of-fifths transformation of the top row and therefore embeds W$_9$ = h3p55 and W$_1$ = h5p33. The remaining rows and columns are T$_0$/T$_4$/T$_8$ transformations of these two. [(22)]

[29] **Example 10b** realizes the matrix for three pianists, one staff/hand/register for each matrix row and one quarter-note beat for each column. Instead of realizing the entire matrix at once, measure 1 articulates the upper-right portion of the matrix (including the main diagonal) and measure 2 the lower left (also including the main diagonal), so that each staff begins and ends with the dyad from the main diagonal, as with the top row's {0, 1}, the second row's {0, 5}, and so forth. As a result, the full columnar aggregate appears only twice, at the end of measure 1 and the beginning of measure 2.

Part IV. MORRIS+$_w$: Other Voice-Leading Possibilities

[30] Part IV considers four larger sets of possibilities involving the same four tetrachord types, each of which embraces all twenty-four ways to transform a given set into another member of the same set class and all twenty-four VL for each transformation. These greatly expanded spaces are labeled MORRIS+$_1$, MORRIS+$_2$, MORRIS+$_4$, and MORRIS+$_5$. The discussion first introduces the additional schritt and wechsel transformations (with one VL for each) and then addresses the multiple VL possibilities.

[31] **Example 11a** presents W$_w$, W$_0$, and W$_{2w}$ within each MORRIS+$_w$. W$_w$ features a VL in which the pcs of the diminished triad move by ci w and the other pc by $-w$, so that all four pcs move by ic w (that is uic = w^4). See, for example, MORRIS+$_2$ W$_2$ 222A (uic 2^4). The total pc content of each W$_w$ transformation is an octatonic collection. Each W$_0$ features a VL that holds the diminished triad invariant and moves the other voice by ci $-2w$; for instance, MORRIS+$_5$ W$_0$

0002 sustains the pcs of the diminished triad while the other pc articulates ci 2. Conversely, each W_{2w} features a VL that moves the diminished triad by ci $2w$ and sustains the other pc, as with MORRIS+$_5$ W_A AAA0, which moves the pcs of the diminished triad by ci A while the other pc is sustained. The total pc contents of W_0 and of W_{2w} are abstract complements of one another, as in MORRIS+$_5$, where they articulate 5-8[02346] and 7-8[0234568], respectively. **Example 11b** presents the remaining trio of wechsel transformations within each MORRIS+$_w$: W_{3+2w}, W_{6+2w}, and W_{9+2w} (e.g. MORRIS+$_4$ W_B, W_2, and W_5). Note that each MORRIS+$_w$ connects to each of the three others via transformations that share the same total octachordal pc content. For example, MORRIS+$_1$'s W_5, W_8, and W_B share 8-23[0123578A], 8-24[0124568A], and 8-20[01245789] with MORRIS+$_4$ W_5, MORRIS+$_5$ W_4, and MORRIS+$_2$ W_1, respectively.

[32] The remaining nine transformations within each MORRIS+$_w$ are the schritt transformations other than S_3, S_6, and S_9, which are easy to envision: transpositionally-related chords in which all voices move in parallel motion by ci n.(23) (S_0 is the identity transformation, which transforms a set onto itself.) These schritt transformations articulate a few interesting features concerning total pc content. First, S_w and S_{-w}, the only schritt transformations with octachordal total pc content, generate the complement of the MORRIS+$_{|2w|}$ tetrachord. When w = 1 or 5, $|2w|$ = 2, which means that MORRIS$_1$'s S_1 and S_B and MORRIS+$_5$'s S_5 and S_7 each create 8-18[01235689], the complement of MORRIS+$_2$'s 4-18. Similarly, when w = 2 or 4, $|2w|$ = 4, so that MORRIS+$_2$'s S_2 and S_A and MORRIS+$_4$'s S_4 and S_8 generate 8-13[01234679], the complement of the MORRIS+$_4$'s 4-13. The latter case is special because pairs of 4-13 tetrachords are generating their own complement. Second, for w = 1, 2, and 5—but not 4 because of the special case just mentioned—S_{2w} and S_{-2w} generate the same total septachordal pc content as W_{2w}. For example, MORRIS$_1$'s S_2, S_A, and W_2 each generate 7-34[013468A]. All in all, it will be helpful to divide the twenty-four transformations within each MORRIS+$_w$ into six groups as follows: The S_0-group includes S_0, S_3, S_6, and S_9; the S_w-group includes S_w, S_{3+w}, S_{6+w}, S_{9+w}; and the S_{-w}-group includes S_{-w}, S_{-3-w}, S_{-6-w}, and S_{-9-w}. The W_0-group includes W_0, W_3, W_6, and W_9, the W_w-group includes W_w, W_{3+w}, W_{6+w}, and W_{9+w}, and the W_{2w}-group W_{2w}, W_{3+2w}, W_{6+2w}, and W_{9+2w}.(24)

[33] With all of the S_n/W_n transformations in place we now consider multiple VL possibilities for each, which, as outlined in part I, can be derived from T-matrices. **Example 12a** provides generalized matrices in terms of n and w (for w = 1, 2, 4, or 5), one for S_n and one for W_n, which allow general properties of the matrices and resulting VL to emerge. For example, each S_n matrix features ten different ci, four that appear multiple times and six others that appear only once. Each W_n matrix features eight different ci, one that appears once, one that appears three times, and six others that appear twice each. Each W_n matrix is symmetrical around the main diagonal. Many consistent patterns may be drawn from the matrices but I will mention only three. First, in every W_n matrix there are four sets of matrix positions that yield an oci with two instances each of two different ci (uci type x^2y^2). For instance, with the matrix for MORRIS+$_w$ W_n the set of positions {(1, 1), (2, 4), (3, 3), (4, 2)} yields oci xyxy, with uci x^2y^2 where x = 6+n and y = 6+n−w.(25) Second, the upper-left 3 × 3 square of a W_n matrix depends on n but not w, which means that all four W_1 matrices have an identical upper-left 3 × 3 square. Third, transformations in the same group feature the same set of ci in these w-independent matrix positions, a set equal to their subscripts taken as a set. For example, the W_0-groups have ci 0, 3, 6, and 9 in their upper-left 3 × 3 squares. These features can be verified by consulting **Example 12b**, which lists all ninety-six matrices.

[34] I will now organize the voice-leading possibilities within this expanded universe by uic type. VL articulate one of five uic types, $|x|^4$, $|x|^3|y|^1$, $|x|^2|y|^2$, $|x|^2|y|^1|z|^1$, and $|x|^1|y|^1|z|^1|q|^1$, all but the fourth of which are addressed below. The discussion of each uic type is in two parts. The first addresses the uci types that generate the uic type and the transformations and VL that create each uci type. The second identifies the possible uic within the uic type, lists the VL within each uic and groups the various uic into categories based on the presence/absence of ics 0, 3, and/or 6. Different uic within the same category contain the same (or similar) numbers of VL, which articulate the same (or similar) number and type of schritt/wechsel transformations. The discussion outlines the organization of the possible VL and suggests several compositional applications.

[35] The MORRIS+$_w$ system includes 2,304 VL. Of these, eighty articulate uic type $|x|^4$, 136 articulate uic type $|x|^3|y|^1$, 432 uic type $|x|^2|y|^2$, 904 uic type $|x|^2|y|^1|z|^1$, and 752 uic type $|x|^1|y|^1|z|^1|q|^1$.

[36] The eighty VL that articulate uic type $|x|^4$ arise from three uci types: x^4, x^3y^1 where x = –y, and uci x^2y^2 where x = –y. Forty-eight VL articulate uci type x^4; these are the straightforward cases involving parallel voice leading. Eight VL arise from uci type x^3y^1 where x = –y: one VL from W_w and one from W_{6+w} within each MORRIS+$_w$ (e.g. MORRIS+$_1$'s W_1 111B and W_7 7775). The other twenty-four VL have uci type x^2y^2 where y = –x. Each MORRIS+$_w$ includes S_3 9393 and S_9 3939, a total of eight VL with uci 3^29^2 and uic 3^4. (The oci is identical for each MORRIS+$_w$ because it involves array positions (3,1), (2,2), (1,3) and (4,4), whose values do not depend on the pc that differentiates the tetrachords from one another.) The remaining sixteen VL occur within the W_{2w}-groups of MORRIS+$_2$ and MORRIS+$_4$. In MORRIS+$_2$, W_4 contains B11B, 4848, and 7755, which produce uic 1^4, 4^4, and 5^4, respectively. W_A also articulates three such VL, and W_7 and W_1 one each. In MORRIS+$_4$ it is W_B and W_5 that include three each and W_8 and W_2 that have one. **Example 13a** lists all of these VL and **Example 13b** provides realizations of some of them in staff notation.

[37] **Example 13c** divides the seven uic (0^4, 1^4, 2^4, 3^4, 4^4, 5^4, 6^4) into three categories.[26] The first category includes uic 0^4 and 6^4, each of which includes only four VL. The second category includes only uic 3^4, which includes sixteen VL, the Z-pairs S_3 3333/9393 and S_9 9999/3939 in each MORRIS+$_w$. Category 3 uic 1^4, 2^4, 4^4, and 5^4 each contain fourteen VL. In uic 1^4 and 5^4 these are distributed 3+5+3+3 across the four MORRIS+$_w$ spaces and in uic 2^4 and 4^4, 2+4+6+2.

[38] **Example 13d** provides a seven-chord, six-voice model created from the MORRIS+$_4$ VL within uic 2^4, with the top four voices articulating one string of VL and the bottom four voices another. Such models are easy to generate because any two voices saturated with ic 2 that do not form harmonic ic 4 (the ic that 4-13 does not contain) can participate in multiple statements of these VL.[27] This model demonstrates variety in several respects. It incorporates all six MORRIS+$_4$ VL with uic 2^4 (S_2 2222, S_A AAAA, W_5 A22A, W_8 2A2A, W_A AAA2, and W_B 22AA) and mixes prime and inverted sets in various ways. At times it unfolds the same VL in both layers simultaneously while at other times the VL differ. The voice leading from one chord to the next usually mixes pc intervals 2 and A, but the third and fourth chords articulate inverse-related VL, S_2 2222 and S_A AAAA, which, since they are applied to sets of contrasting orientation, create pc interval 2 in all voices. Moreover, the seven chords articulate six different set types; each of the first six chords articulates a MORRIS-constellation hexachord because it is comprised of two 4-13s that share two pcs, and the final chord has only four pcs—the same 4-13 in both layers. **Example 13e** provides a musical realization based on the model: the middle two voices of the model appear in the cello part, a mixture of double stops and compound melody, and the other model voices appear in the piano part, all in the treble register. The middle of the excerpt articulates the complete model, while the beginning and end offer fragments.[28]

[39] There are 136 VL with uic type $|x|^3|y|^1$; forty derive from uci x^3y^1 where $y \neq -x$ and ninety-six from uci = $x^2y^1z^1$ where z = –x. Each W_n has precisely one VL in which the diminished triad moves by ci n and the other pc moves by ci n–2*w*, which creates forty-eight uci of the form x^3y^1 (where y = n–2*w*). Omitting the eight instances where y = –x leaves forty VL with uic = $|x|^3|y|^1$. Half of the remaining ninety-six VL articulate S_n, six for each of eight values of n (1, 2, 4, 5, 7, 8, A and B). Considering n = 1 as an example, MORRIS+$_1$ S_1 B113, MORRIS+$_4$ S_1 1B13, and MORRIS+$_5$ S_1 311B generate uic 1^33^1; MORRIS+$_2$ S_1 7975 generates uic 5^33^1; and MORRIS+$_4$ S_1 4480 and S_1 2AA6 generate uic 4^30^1 and 2^36^1, respectively. Another sixteen arise because W_3, W_9, W_{w+3}, and W_{w+9} articulate one VL of this type within each MORRIS+$_w$. These VL involve voice exchange, indicated by *. For example, MORRIS+$_5$ W_3 9395* can be realized by the set of voices {0–9, 3–6, 6–3, 2–7}, in which 3–6 and 6–3 articulate a voice exchange. Accounting for the remaining thirty-two VL, each transformation within the W_{2w} group of MORRIS+$_1$ and MORRIS+$_5$ articulates four VL with uic $|x|^3|y|^1$. For instance, MORRIS+$_5$ W_A includes AA82, 2A42, 2AA8, and 4A88; the first three articulate uic 2^34^1—forming a *Z-triple*—and the last one uic 4^32^1. **Example 14a** provides a complete list of VL with uic type $|x|^3|y|^1$ and **14b** realizes the S_1 and MORRIS+$_5$ W_A VL in musical notation.

[40] **Example 14c** organizes all 136 VL with uic type $|x|^3|y|^1$ by uic. There are four categories. Category 1 includes the four uic for which $|y| = 3 \times |x|$: these are 1^33^1, 2^36^1, 4^30^1, and 5^33^1. Each of these four uic includes twelve S_n-based VL along with two or four W_n-based VL. The remaining three categories include only wechsel-derived VL. Each uic in category 2 includes eight VL, each one in category 3 twelve VL, and each one in category 4 only two.

[41] The 432 VL with uic type $|x|^2|y|^2$ articulate one of three uci types: uci type x^2y^2 where $x \neq -y$ (216 VL), uci type $x^2y^1z^1$ where $z = -y$ (136 VL), and uci type $x^1y^1z^1q^1$ where $x = -z$ and $y = -q$ (80 VL).

[42] Of the 216 VL that articulate uci type x^2y^2 where $x \neq -y$, forty are S_n (6+n)n(6+n)n where n ≠ 3 or 9—S_0 6060*, S_1 7171, S_2 8282, S_4 A4A4, and so forth. The remaining ones arise from W_n transformations. Most MORRIS$+_w$ W_n articulate four such VL, one for each of the two dyad partitions that involves [03] and two for the partition that involves [06].[29] For example, MORRIS$+_5$ W_4 features 8118, 7722, A5A5, and 4545, which articulate uci 1^28^2, 2^27^2, 5^2A^2, and 4^25^2, and uic 1^24^2, 2^25^2, 2^25^2, and 4^25^2, respectively. In 8118, the embedded [03] dyad formed by sm 2 and sm 3 moves by ci 1 and the [02] dyad formed by sm 1 and sm 4 moves by ci 8; and in 7722, the [03] dyad formed by sm 1 and sm 2 moves by ci 7 and the [04] dyad (sm 3 / sm 4) moves by ci 2; in A5A5 and 4545 the [01] dyad (sm 2 / sm 4) moves by ci 5 and the [06] dyad (sm 1 / sm3) moves by ci A in one case and by ci 4 in the other. The upper portion of **Example 15a** lists all of these VL. In **Example 15b** the VL mentioned in the discussion are realized so that the top two voices move in parallel motion by one ci and the bottom two voices move in parallel motion by another.[30]

[43] The 136 VL with uci type $x^2y^1z^1$ where $z = -y$ arise as follows: each MORRIS$+_w$'s S_0 and S_6 include six such VL, each S_3 and S_9 includes four, each W_w and W_{6+w} three, and each W_3, W_9, W_{3+w} and W_{9+w} two. Each S_0 includes five VL that hold two pcs (ci 0) while the other two voices articulate a voice exchange (*) and one VL with a double voice exchange (**) (e.g. MORRIS$+_2$ S_0 0057*, 3900*, 0804*, B001*, 0390*, and 6864**), each S_6 includes one VL with a pair of ic 0 and five with a pair of ic 6 (e.g. MORRIS$+_2$ S_6 020A, 5667, 66B1, 9366, 6936, 626A*), and so forth. The middle part of the chart in Example 15a provides the complete list and **Example 15c** realizes these and other sample VL in musical notation.

[44] The 80 VL with uci type $x^1y^1z^1q^1$ where $z = -x$ and $q = -y$ arise only from the S_0 group of each MORRIS$+_w$ and from the W_{2w} group of MORRIS$+_2$ and MORRIS$+_4$. Each S_0, S_3, S_6, and S_9 includes two VL, both of which involve ic 3 (e.g. MORRIS$+_5$ S_0 3984** and 239A**, S_3 5397* and 93B1, S_6 932A and 8934, S_9 3957*and B931). Each W_{2w}-group transformation includes six VL organized into three Z-pairs; for instance, MORRIS$+_2$ W_4 includes 715B/B715 (uic = 1^25^2), B148/481B (uic = 1^24^2), and 4758/7845 (uic = 4^25^2). The lower part of Example 15a includes the complete list and **15d** lists most of the cases just mentioned in staff notation.

[45] **Examples 15e** and **15f** illustrate a compositional application involving one of these Z-pairs: MORRIS$+_2$ W_4 715B/B715. Example 15e includes four two-chord, four-voice models, labeled X 715B, Y 715B, X B715, and Y B715. X and Y denote tetrachord pairs (X = {0, 3, 6, B}–(A, 7, 4, B} and Y = {1, A, 7, 2}–(3, 6, 9, 2}) and 715B and B715 signify the motion of the voices. Each model is arranged so that pc interval B appears in the soprano register, pc interval 1 in the alto, pc interval 5 in the tenor, and pc interval 7 in the bass. Example 15f provides a short musical excerpt for piano that includes five different combinations of these models, in which voices sometimes appear an octave higher or lower than in the model in order to achieve more sonorous spacing and/or to avoid registral overlap. The excerpt first explores the four ways to combine either X 715B or X B715 with either Y 715B or Y B715. Although each combination results in the same pair of octachordal pcsets and the same melodic motion within each register, there is considerable variety because each combination creates a different pair of parallel dyads within each register, as with the soprano's {B5, G6}–{A♯5, F♯6}, {B5, B♭6}–{A♯5, A6}, {C7, G7}–{B6, F♯7}, and {C6, B♭6}–{B5, A6}, which articulate harmonic ics 4, 1, 5, and 2, respectively. The excerpt ends with the simultaneous unfolding of X 715B, X B715, Y 715B, *and* Y B715, sixteen-voice harmony in which each pc of X and Y appears in two registers. Each register contains parallel tetrachords, 4-4[0125] in soprano and alto and 4-14[0237] in bass and tenor.

[46] **Example 15g** lists all of the VL of uic type $|x|^2|y|^2$, organized by uic. The twenty-one possible uic organize into six categories. Category 1 involves only ics 0, 3, and/or 6, that is, uic 0^23^2, 0^26^2, and 3^26^2. All VL within these uic arise from S_0-group transformations and have identical oic in each MORRIS$+_w$. Category 2 involves ic 0 but not ics 3 and 6. Each uic 0^2n^2 (0^21^2, 0^22^2, 0^24^2, 0^25^2) includes fourteen VL: nine VL that constitute the MORRIS$_n$ VL discussed at length in parts II and III of the paper, one S_0 VL in each of the other three spaces, and a further pair in MORRIS$+_{6-n}$. Category 3 involves ic 6 but not ics 0 and 3. Mirroring category 2, each uic 6^2n^2 (6^21^2, 6^22^2, 6^24^2, and 6^25^2) embeds fourteen VL: nine within MORRIS$+_{6-n}$, one S_6 VL in each of the other three spaces, and a further pair in MORRIS$+_n$.

[47] The fourth category involves ic 3 but not ics 0 and 6. Each uic 3^2n^2 (3^21^2, 3^22^2, 3^24^2, and 3^25^2) embeds thirty-two VL, ten within MORRIS$_n$, ten within MORRIS$_{6-n}$, and six in each of the other two spaces. A fascinating subset of each uic in this category is the presence of four VL pairs that share not only melodic ics but harmonic ones. Consider as an example MORRIS$+_2$ W_2 9BB9 and MORRIS$+_4$ W_4 9119. In each case, sm 2 and sm 3 create ic 3 harmonically and move by ic 1 melodically, and sm 1 and sm 4 create ic 1 harmonically and move by ic 3 melodically. **Example 15h** realizes this pair and the three other pairs within uic 1^23^2.

[48] Categories 5 and 6 do not involve ics 0, 3, and 6. In category 5, one ic is odd and the other even (1^22^2, 1^24^2, 2^25^2, and 4^25^2) and in category 6 both are odd or both are even (1^25^2 and 2^24^2). Within category 5, each uic includes twenty VL, all of which articulate wechsel transformations. Within category 6, each uic consists of thirty-six VL, including a *pair of Z-quadruples* (e.g. MORRIS$+_2$'s W_1 4A28/84A2/4422/8AA8 and W_7 A482/2A48/2442/AA88 within uic 2^24^2).

[49] The discussion of uic type $|x|^2|y|^2$ concludes by pointing out a few such VL within an excerpt from Bartok's String Quartet No. 6. The excerpt in **Example 16** is the opening of the piece, in which the outer voices present overlapping statements of the opening 3½-measure melody (bracketed on the score) and then diverging sequences based on fragments of that melody. The inner voices are also primarily stepwise, moving first in free counterpoint and then in parallel tritones. Harmonically, the excerpt features five triadic anchor points, nineteen instances of the MORRIS tetrachords, and a variety of other sonorities. The MORRIS tetrachords create six VL of uic type $|x|^2|y|^2$: two from MORRIS$_1$ (W_9 00BB and W_7 1010), one from MORRIS$_2$ (S_3 200A), a side-by-side pair from MORRIS$+_4$ (S_B 5B5B and W_A A0A0), and one from MORRIS$+_5$ (S_0 6B61**). Two of these VL articulate uic 0^21^2, two uic 0^22^2 and the others 1^25^2 and 1^26^2. Four involve ic 0, which is not surprising given the frequency with which two voices hold while two others move, and all six involve either ic 1 or ic 2, which makes sense given the stepwise thematic material. In all but two cases "passing sonorities" complicate the musical realizations. The rhythmic offset of the moving voices of MORRIS$_2$ S_3 200A creates an intervening 4-17[0347], parallel tritones moving by half step decorate W_7 1010 with a pair of 4-16s [0157], and further stepwise motion during MORRIS$+_4$ S_B 5B5B and MORRIS$+_5$ S_0 6B61** creates embellishing 4–5 [0126] and 4–6 [0127] sonorities. Moreover, each VL features precisely *two voices* moving stepwise (either by ic 1 or ic 2), which creates contrast with the approaches to the F♯m, GM, E♭M and Cm triadic anchors, each of which involves stepwise motion in *all four voices*. (The approaches to F♯m and GM mix half- and whole-step voice leading and the approaches to E♭M and Cm feature all four voices moving by half step.) Finally, taken as a group, the roots of all five triadic anchors (A, F♯, G, E♭, C) articulate 5-31 [01369], the sole pentachordal set type that embeds all four of the MORRIS tetrachords.

[50] There are 752 VL that articulate uic type $|x|^1|y|^1|z|^1|q|^1$, all of which derive from uci type $|x|^1|y|^1|z|^1|q|^1$ where $x \neq -y$, $x \neq -z$, $x \neq -q$, $y \neq -z$, $y \neq -q$, and $z \neq -q$. Each S_n and W_n generates between 20 and 48 VL of this uic type (consult **Example 17a**) and the VL organize into nineteen uic (which divide into six categories) as shown in **Example 17b**. The reader may wish to step through these charts on their own as I have done with similar charts above, or simply proceed with the discussion here, which provides a compositional application involving one sample uic and then highlights another special uic.

[51] VL with uic type $|x|^1|y|^1|z|^1|q|^1$ can be used to differentiate voices from one another. As an example, consider the twelve MORRIS$+_4$ VL with uic $0^12^13^15^1$: S_0 0372/0A95, S_3 90A5, S_9 0732, W_0 0295/5902, W_3 9025/9502, W_A 37A0/A073, and W_1 73A0/7A03. **Example 17c** realizes these in staff notation so that ic 0 always appears in the soprano voice, ic 2 in the alto, ic 3 in the tenor, and ic 5 in the bass. **Example 17d** constructs a short excerpt for string quartet that incorporates three $0^12^13^15^1$ VL. The passage's six 4-13 chords articulate S_9 0732–(S_9 3669)–W_0 0295–(W_7 777B)–S_0 0372. For each $0^12^13^15^1$ VL, ic 0 appears in violin I, ic 2 in violin II, ic 3 in viola, and ic 5 in the cello. Since every second VL is part of $0^12^13^15^1$, each pc in the passage participates in a statement of its instrument's primary ic; for instance, violin II's D♭–E♭, A–B, and C–D state ic 2. The parenthesized VL, which do not articulate uic $0^12^13^15^1$, arise as a result of the consistent ic/instrument pairing within the $0^12^13^15^1$ VL. Three other features of the passage bear mentioning. First, the $0^12^13^15^1$ VL involve more shared pcs as the passage progresses; that is, at S_9 0732 the 4-13s share two pcs, at W_0 0295 three, and at S_0 0372 all four. This is reflected by gradual decreases in dynamic level, rhythmic activity, and register. Second,

all 4-13s but the fourth are in prime orientation; these articulate S_9–S_9–S_5–S_0, which is a large-scale projection of 4-13. Finally, the staggered pc entrances within each 4-13 are calculated to reinforce pitch/rhythmic relationships; that is, each ic 2 in violin II articulates an attack-point duration of 2 (taking the eighth note as the unit), each ic 3 in the viola duration 3, and each ic 5 in the cello duration 4. The simultaneous arrival of the pcs in the final chord strengthens the sense of cadence.

[52] We conclude the discussion of uic type $|x|^1|y|^1|z|^1|q|^1$ by pointing out uic $1^1 2^1 4^1 5^1$, whose sixty-four VL are the most of any uic we have seen. These sixty-four organize into sixteen Z-pairs, with a further *quartet of Z-quadruples* in both MORRIS$+_2$ and MORRIS$+_4$. **Example 17e** realizes the twenty MORRIS$+_2$ VL within this extraordinary uic.

Part V. Concatenating MORRIS+*w* VL and Controlling Individual Voices

[53] Part V focuses on several ways to concatenate MORRIS$+_w$ VL while controlling the content of one or more individual voices. First, it concatenates repeated instances of a single VL and alternates Z-related VL. Second, a set of VL within a single uic are put together so that they saturate each voice with a different ic and then so that each voice alternates between two ics. Two further examples saturate a given voice with a particular pc interval (not just interval class) and another shows how to maintain a consistent relationship between two voices.

[54] In order to study the content of individual voices it is crucial to consider the following general feature of VL concatenation: given a series of three chords that articulate VL_a then VL_b, ci x_a at matrix position (r_a,c_a) within the matrix of VL_a, and ci x_b at matrix position (r_b,c_b) within the matrix of VL_b, *ci x_b will follow ci x_a in the same voice if and only if $c_a = r_b$.*

[55] For example, if VL_a and VL_b are both equal to MORRIS$+_2$ S_2 8273, ci 8 at matrix position (1,3) within VL_a will be followed in the same voice by ci 7 at matrix position (3,4) because the column of (1,3) and the row of (3,4) have the same value, 3. Similarly, ci 7 at matrix position (3,4) within VL_a will be followed in the same voice by ci 3 at matrix position (4,1) within VL_b, ci 3 at matrix position (4,1) within VL_a will be followed in the same voice by ci 8 at matrix position (1,3) within VL_b, and ci 2 at matrix position (2,2) within VL_a will be followed in the same voice by another statement of ci 2 within VL_b. Not only does this information give a complete description of the voices for the three chords that articulate VL_a–VL_b, but it also dictates that additional consecutive applications of MORRIS$+_2$ S_2 8273 will result in three voices saturated with the recurring ci pattern 8–7–3–... and the other voice with repetitions of ci 2. Three consecutive statements of S_2 8273 yields an overall S_6 6666 (because 8 + 7 + 3 = 2 + 2 + 2 = 6) and six consecutive statements creates an overall S_0 0000, a return to the original chord with the same pcs in the same voices. The four-voice realization in **Example 18a** illustrates this. The bass, soprano, and tenor voices are saturated with ci-succession 8–7–3. The bass voice is composed of two statements of it (C♯–A–E–G and G –E♭–B♭–C♯) and the soprano and tenor voices are transposed rotations of the bass voice. Differentiating itself from the other voices, the ci-2 saturated alto voice articulates a complete whole-tone scale.

[56] **Example 18b** provides a slightly different situation: twelve consecutive statements of MORRIS$+_2$ S_2 5A89 saturate *all four voices* with a repeating ci pattern, 5–A–9–8, which creates S_8 8888 every fourth chord and an overall S_0 0000. **Example 18c** alternates the Z-related VL that appear separately in Examples 18a and 18b. The result is a pair of voices saturated with ci-succession 3–5–2–A (soprano imitated two beats later at T_4 by alto), another voice that alternates ci 7 and 9 (bass), and another that repeats ci 8 (tenor). Overall, these sequential patterns create voices that articulate symmetrical pc collections: the augmented triad (3-12[048]), the whole-tone, hexatonic, and enneatonic collections (6-35 [02468A], 6-20[014589], 9-12[01245689A]), and 6-30[013679], which is invariant under T_6.

[57] There are a few notable differences when wechsel transformations are involved. First, by definition, the juxtaposition of a VL with its inverse creates an overall S_0 0000, a simple neighbor-like return to the original chord as illustrated by MORRIS$+_1$ W_B 28B9 and B289 in **Examples 18d** and **18e**. Second, consecutive statements of a single W_n create an oscillation between two chords, as with **Example 18f**'s series of W_B 28B9, which alternates $D^{ø7}$ and A^7. Highlighting chordal identity, the musical realization places all four voices in the same register so that each $D^{ø7}$ features the same set of pitches as does each A^7. Third, any series of wechsel transformations causes an alternation between prime and inverted orientations of the chords, which means that if a voice is saturated with a single ci there is an alternation of inversionally-related pc intervals and therefore an oscillation between two pcs. For example, the series of W_B 28B9 in Example 18f creates

a soprano line saturated with ci 9, which causes an alternation between pc intervals 9 and 3 articulated by C–A–C–A– . . . Indeed, any repeating pattern involving an odd number of ci creates abutting inversionally-related segments. For instance, Example 18f's bass voice features ci succession 2–8–1 twice, once articulated by D–E–A♭–G (pc intervals 2–4–B) and once by G–F–C♯–D (pc intervals A–8–1). Fourth, rotationally-related voices in these wechsel series are related to one another by rotation and $\underline{T_0}$, as with the tenor and alto voices which imitate the bass at T_0, two and four chords later, respectively. Finally, Z-triples and quadruples can be used to generate interesting examples such as 18f (and avoid brief neighbor-like figures such as 18d and 18e) as long as inverse VL are not placed side-by-side. For instance, one could arrange the VL in the Z-triple MORRIS+$_5$ W_1 11B5/511B/71BB into a recurring pattern in which statements of 71BB separate the inverses 11B5 and 511B from one another: 11B5–71BB–511B–71BB.

[58] The general feature of VL connection can also be used to create voices consisting of a single ic. For instance, given the twenty MORRIS+$_2$ VL within uic $1^1 2^1 4^1 5^1$, it is possible to concatenate VL so that each voice is limited to a single ic. **Example 19a** graphs all such connections. The graph is in two disjoint but congruent halves, dividing the twenty VL into two groups of ten. Single-direction arrows identify situations where one VL may follow another (e.g. W_1 15A8 may follow W_4 78AB) and double-direction arrows identify (inverse-related) VL that may precede or follow one another (e.g. W_1 15A8 and W_1 8A15). Twelve VL have precisely two VL that could precede it and two that could follow it, four others have two VL that could precede it but only one that could follow it, and the remaining four have only one VL that could precede it but two that could follow. **Example 19b** provides one path through the left half of the graph, along with a chart that shows that, for each ic, the matrix column value for one VL is the same as the row value for the next.[31] **Example 19c** provides a four-voice model that articulates the path given in Example 19b. **Example 19d** provides a short musical excerpt based on the model. The ic-1 voice appears in the flute, the ic-5 voice in the clarinet, and the ic-2 and ic-4 voices in the vibraphone dyads.

[59] Using the same set of VL, it is possible to create alternation between two ics within each voice. The graph in **Example 20a** illustrates how to concatenate VL so that two voices alternate ics 1 and 2 and the other two alternate ics 4 and 5. The graph is radically different in shape from the previous one: four VL are dead ends and four others are unapproachable beginnings (these eight appear in the center of the graph), and the remaining twelve each have two possible antecedents and two possible consequents that create a complex web with no double-direction arrows. **Example 20b** provides a four-voice realization for one nine-VL path through the graph and **20c** lists ci and matrix position connections for each voice.

[60] We now consider how to control pc interval (not just interval class) within an individual voice. Since having twenty VL within the same uic and MORRIS+$_w$ is a rare luxury, we study a situation with fewer possible VL, the set of eight MORRIS+$_4$ schritt transformations that articulate uic $0^1 1^1 2^1 5^1$ (S_1 7BA0, S_B 0251, S_2 20B7, S_2 5021, S_A A150, S_A B7A0, S_5 B207, and S_7 A510). With these eight VL, even the seemingly modest goal of saturating a single voice with pc interval 5 reaches a dead end after, at most, three VL. One reasonable solution is to permit the featured voice to articulate pc interval 0, but only whenever continuation by pc interval 5 is impossible. The graph in **Example 21a** illustrates the possibilities for VL concatenation assuming the use of chords in prime orientation. Since the featured voice may contain side-by-side ci 5 but not side-by-side ci 0, it is necessary to track whether the featured voice is articulating ci 5 (e.g. S_A A1**5**0) or ci 0 (e.g. S_A A15**0**).[32] **Examples 21b–d** provide musical realizations for three paths within the graph, focusing on S_A A150 and S_7 A510 and avoiding dead-end nodes such as S_2 5**0**21. These series articulate overall S_A AAAA, S_8 8888, and S_6 6666, respectively, which means that sequential repetition of each string of VL would eventually lead to an overall S_0 0000. **Example 21e** shows the matrix position connections for the featured voice of the path in 21b.

[61] Of course if VL and featured pc interval are carefully chosen, it is possible to saturate a voice with one pc interval with only a few VL and without resorting to interspersed pc interval 0. For example, MORRIS+$_4$'s W_B B1B1, W_7 AA1B, and W_8 1B2A can be combined to create an infinitely long series of pc interval B in one voice—no matter the initial chord's orientation or featured-voice sm. If the initial chord is in prime orientation ci B will create pc interval B; if sm 1 is in the featured voice then W_B **B**1B1 is needed, if sm 2 then W_8 1**B**2A, if sm 3 W_B B1**B**1, and if sm 4 W_7 AA1**B**. If the initial chord is in inverted orientation ci 1 will create pc interval B; if sm 1 is in the featured voice then W_8 **1**B2A is needed, if sm 2 then W_B B**1**B1, if sm 3 W_7 AA**1**B, and if sm 4 W_B B1B**1**. **Example 22a** presents a graph showing how the string of pc B

would be continued. The graph is somewhat strange because any starting point leads, sooner or later, to the same (potentially endless) circuit of four nodes. The ability to generate an infinite series of pc interval B with so few VL is a result of the multiple occurrences of ci 1/B within the VL and the freedom to choose VL with different uic. Added benefits include that W_7 AA1B and W_8 1B2A articulate the same uic ($1^2 2^2$), and that the three VL articulate only two different ics (1 and 2), which saturates the texture with stepwise motion. **Example 22b** provides a realization of one path through the graph, **22c** shows the matrix position connections for the voice that articulates pc interval B, and **22d** offers a straightforward musical realization of the model for string quartet.

[62] It is also possible to maintain a consistent relationship between two voices, such as a series of VL that keeps two voices ic 3 apart, creating parallel motion. **Example 23a** lists the thirty-seven MORRIS+$_5$ VL for which two voices that are ic 3 apart proceed by ci 0, 1, 2, A, or B. In all such VL the first two or middle two ci of the oci are the same (e.g. W_9 1195, W_0 400A, W_5 B227). These thirty-eight VL are divided into four boxes based on the matrix positions of the repeated ci: in the upper left box positions (1,2)/(2,1), in the upper right (2,3)/(3,2), in the lower left either (1,3)/(2,2) or (1,2)/(2,3), and lower right (2,2)/(3,1) or (2,1)/(3,2). The method of connection is the same as in previous examples, that is, to keep the parallel ic 3 within the *same pair of voices* the column values for one VL's repeated ci must match the row values for the next. For example, the repeated ci of VL in the upper left box appear in columns 1 and 2 and so they must be followed by VL whose repeated ci appear in rows 1 and 2, that is, by a VL from that same box or by one from the lower left box. **Examples 23b–23e** demonstrate a few possible ways to concatenate these VL. In 23b the upper voices ascend in parallel minor thirds by whole step and in 23c they descend by half step using only schritt transformations. In Example 23d, the lower voices oscillate between two major sixths, {E♭3, C4} and {F3, D4}, in an unpredictable rhythm caused by VL featuring pc interval 0, while the upper voices explore the A major diatonic collection. In Example 23e, parallel minor thirds articulate a repeating pattern of pc intervals, 0–B–A–0–B–A–0.

[63] **Examples 23f** and **23g** provide musical passages based on the models in 23c and 23e, respectively. Example 23f features a single melodic line for flute that incorporates all four voices of the model (transposed up a major third). Example 23g, for piano six hands, creates a series of twelve-tone chords that includes the four-voice model of Example 23e unfolding simultaneously with its T_7 duplicate; this creates a series of 8-18[01235689] octachords over which is superimposed the series of 4-18[0147] that completes each vertical aggregate. Player II plays a series of parallel perfect fifths that articulate the soprano voice of the model and its T_7 relative (right hand) and a series of parallel perfect fourths that articulate the alto voice of the model and its T_7 relative (left hand). Player III treats the tenor and bass voices of the model in a similar fashion. Complementing this MORRIS+$_5$–based music and completing the twelve-tone structure, player I plays a MORRIS+$_2$ series (with a few passing and neighbor tones) that emphasizes voice-leading ics 0, 1, and 2.

[64] This paper defines contextual intervals (ci), ordered/unordered lists of contextual intervals and interval classes (oci, uci, uic), voice-leading transformations (VL), and related concepts, and uses them to identify and organize the one-to-one and onto voice-leading possibilities for each of four tetrachordal set types. In this context, it presents the MORRIS$_w$ VL possibilities suggested by Soderberg 1998, demonstrates a few applications, expands the universe to create MORRIS+$_w$, and shows various ways to concatenate VL while controlling the motion of one or more individual voices. It also categorizes VL in several ways and suggests a variety of compositional applications, in which four-, six-, and twelve-voice pitch-class models yield various monophonic, homophonic, and contrapuntal musical textures, within octatonic, twelve-tone, and other pitch contexts.

[65] A variety of future directions are suggested. There are other ways to define voice-leading spaces within the MORRIS+$_w$ universe, including the following: First, gather together VL with similar (or identical) uic; for example, VL that come from uic $1^3 4^1$, $4^3 1^1$, $1^2 4^2$, $0^2 1^1 4^1$, $1^2 0^1 4^1$, and $4^2 0^1 1^1$ include at least one ic 1 and at least one ic 4, may also include one or two ic 0, and do not include any other ics. Second, limit VL by transformation; for instance, explore concatenations of the forty-eight MORRIS+$_5$ VL that articulate either W_1 or W_4. Third, explore situations where one or more set members move by a particular ic or ci; for instance there are eight MORRIS+$_2$ VL in which sm 1 moves by ci 1 and sm 4 by ci B.[33]

[66] There are also many ways to branch out from the MORRIS+$_w$ universe. For example, I plan to study the

transformations and VL that arise from moving directly from one MORRIS tetrachord to another; for example, a given member of 4-18 can be followed by any one of twenty-four members of 4-27 and there are twenty-four ways to voice lead each of these, for a total of 576 VL. Such study suggests relating the MORRIS$_w$ tetrachords and resulting chord progressions to (extended) tonal practice.[34] Also, the VL machinery may be applied to other individual set types. Non-symmetrical tetrachords have the same number of VL (576) but symmetrical ones have fewer. Finally, it is possible to connect the MORRIS tetrachords to others, address sets of other cardinalities, and consider VL that are not one-to-one and onto. All of this, along with the potential analytic and compositional applications of these theoretic possibilities, makes it clear that (tetrachordal) voice leading includes an infinitely interesting set of options that we have only begun to explore.

Mark Sallmen
University of Toronto

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Footnotes

1. See the discussion of DOUTH2 in Lewin 1996, as well as Childs 1998, Cohn 1998, Gollin 1998, Douthett and Steinbach 1998, and Hook 2002 & 2007. All of this research concerning 4-27 grows from similar work with major and minor triads, which appears in all of the studies cited above (except Soderberg 1998) and many others, including Hyer 1995, Cohn 1996, 1997, & 2000, Capuzzo 2004, Engebretsen 2008, and Kochavi 2008.

For voice-leading studies that address these and other set types, see Roeder 1989, Straus 1997 & 2003, Callender 1998, Morris 1998, Lewin 1998 & 2001, Alegant 2001, Cope 2002, Cohn 2003, Tymoczko 2005 & 2010, Childs 2006, Callender, Quinn, and Tymoczko 2008, Rockwell 2009, and Waters and Williams 2010.
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2. Since each MORRIS+$_w$ space includes 576 voice-leading transformations, the number of ways to choose one or more of these possibilities is vast: there are 576 (= 576!/(575!1!)) ways to choose one, 165,600 (= 576!/(574!2!)) ways to choose two, 31,684,800 (= 576!/(573!3!)) ways to choose three, and so on. Coupled with this huge number, the potential rules for concatenation involve an infinite number of ways of requiring, preferring, allowing, or forbidding.
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3. The pair of 4-18 tetrachords provides double-neighbor-like ornamentation of a French sixth chord, which is not part of the MORRIS(+)$_w$ universe.
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4. Pairs of 4-27 tetrachords connected by ic 1 in each voice are addressed by Cohn 1998, 295.
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5. See Hook 2002, Lewin 1993, 25–30, Klumpenhouwer 1994, Clough 1998, Kochavi 1998, Roeder and Cook 2006, and Cook 2009. Sallmen 2009 uses schritt and wechsel transformations to study series of 4–18[0147] tetrachords in Elliott Carter's "Dolphin."
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6. Parsimonious voice leading is addressed by the 4-27– and triad–centered literature cited above, as well as by Roeder 1989, Callender 1998, Cohn 2003, Tymoczko 2005 & 2010, Roeder and Cook 2006, and Rockwell 2009.
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7. For T-matrices and voice-leading lists see Morris 1998. For other ways to consider the voice-leading possibilities between two sets consult "IFUNC(X, Y)i" in Lewin 1987 & 2001, "the interval vector between X and Y" in Morris 1987, and the "progression vector" in Nauert 2003. For sample compositional applications of compositional designs, compositional spaces, and voice-leading spaces, consult Morris 1987, 1995, & 1998, respectively.
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8. Morris 1998 calls this definitive R3-type voice leading. For careful consideration of voice-leading types that include pc omission and/or doubling see Morris 1998, 203–206, and Lewin 1998.
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9. See, for instance, Morris 1995 & 1998 and Cohn 2000 & 2003.
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10. The concept of sm is crucial to a detailed discussion of voice leading and a conventional ordering is a clear way to identify sm. This particular choice of sm and conventional ordering highlights consistencies from one tetrachordal space to another because the diminished triad always involves sm 1–3 and the "warped pc" sm 4. It would have been possible to take sm from the familiar set-class prime forms—and I tried this in my first drafts of this work—but such a system conceals inter-space consistency because of the varying placement of the diminished triad. That is, the diminished triad appears as the second, third and fourth digits in [0258] and [0147] but as the first, third and fourth in [0136] and [0236], and it does not help that the diminished triad variously appears as "036," "147" and "258."
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11. The W subscripts in Hook's 2002 study of set class 4-27[0258] reflect the motion of the roots of the major-minor and half-diminished seventh chords, which has the advantage of connecting the W_n to conventional tonal analysis. Hook's W_n corresponds to my MORRIS$+_1$ $W_{(4-n)}$. As examples, C^7–$C^{ø7}$, C^7–$C\sharp^{ø7}$, and C^7–$E^{ø\frac{4}{2}}$ articulate Hook's W_0, W_1, and W_4, respectively, and my MORRIS$+_1$ W_4, W_3, and W_0, respectively.
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12. Oci alone is insufficient to define individual voice leading situations. First, some oci appear in multiple MORRIS$_w$ spaces and we need a way to distinguish these situations from one another. For example oci 9003 is part of four different VL: MORRIS$_1$ S_3 9003, MORRIS$_2$ S_3 9003, MORRIS$_4$ S_3 9003 and MORRIS$_5$ S_3 9003. Second, some oci satisfy the harmonic requirements of only one of the four tetrachord types and there is usually no way quick way to determine this (other than trial and error). For example, when applied to a member of 4-13[0136], oci 8259 produces another member of the same set class, but applied to any other MORRIS$+_w$ set type oci 8259 produces a second chord that is of a different set type and therefore not within the scope of this paper.
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13. Although the notational conventions of my VL and Lewin's 1998 "a voice leading from X into Y" (e.g. {A → G♯, D → E, F → E}) are somewhat different, the concepts share much in common. Both identify—either explicitly or implicitly—the

chord/set members of X and Y involved in each voice and the melodic intervals they form. The primary difference is that all of my VL are one-to-one and onto whereas some of Lewin's voice leadings are not. For an approach that defines voice leading by explicitly listing, within each voice, the set member of X, the set member of Y, and the melodic interval, consult Cope 2002, 125.
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14. For X = {5, 2, B, 4}, a set in inverted orientation, sm 1 is pc 5, which moves by ci 8 to pc 9, sm 2 is pc 2, which moves by ci 2 to pc 0, sm 3 is pc B, which moves by ci 5 to pc 6, and sm 4 is pc 4, which moves by ci 9 to pc 7.
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15. Since the T-matrix features *contextual intervals* (not pc intervals) any pair of sets that articulate the pcset transformation—no matter the orientation—generates the same matrix. For example, X = {6, 3, 0, 5} and Y = {7, A, 1, 8}, which also articulate MORRIS+$_4$ W$_5$, would also generate the matrix in Example 3b, keeping in mind that in this case the contextual intervals are the inverses of the pc intervals.

By definition, all VL in this paper involve each pc in each set exactly once, but the matrix gives *all* voice leading possibilities, including those that would double and/or omit pcs from one or both sets. For instance, with looser restrictions one could consider the set of voices {0–5, 3–8, 6–B, 3–A}, which includes each sm of Y precisely once but which doubles sm 2 of X and omits sm 4 of X and which articulates ci 5 @ (1, 3), (2, 2), and (3, 1), and ci 7 @ (2, 4).
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16. A more detailed explanation of inverses: For each ci at matrix position (j, k) within a given wechsel VL, the inverse VL includes the same ci at matrix position (k, j), which ensures that, if the VL are juxtaposed, both instances of the ci appear in the same voice. Since wechsel transformations alternate prime and inverted orientations, consecutive instances of the same ci in a given voice result in an overall ci 0. For example, ci 8 appears at (1, 2) in W$_5$ 8259 but at (2, 1) in W$_5$ 5829, which means that, in the example, the soprano voice moves by ci 8 from sm 1 (pc 0) of the first chord to sm 2 (pc 8) of the next chord and then by ci 8 back to sm 1 (pc 0) of the third chord. For inverse-related schritt VL, the row/column swapping works the same way but, since schritt transformations do not alternate orientations, inverse transformations and ci are involved.
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17. For more practice with the notions of S$_n$/W$_n$, VL, uic and inverse, the reader may return to the musical excerpts in Example 2, which articulate MORRIS$_2$ S$_6$ 020A (uic $0^2 2^2$), MORRIS+$_1$ W$_1$ 111B (uic 1^4), MORRIS+$_2$ W$_1$ 8AA8 (uic $2^2 4^2$), and MORRIS+$_5$ W$_1$ 73A0 (uic $0^1 2^1 3^1 5^1$), respectively. Example 2c's MORRIS+$_2$ W$_1$ 8AA8 is its own inverse, which means that the chordal alternation in the excerpt articulates side-by-side statements of this VL.
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18. Within each MORRIS$_w$, S$_3$ *w*00(−*w*) and S$_9$ 00*w*(−*w*) are inverses because applying one then the other results in an overall S$_0$ 0000. The remaining VL are involutions because they are their own inverses and so applying the VL twice yields an overall S$_0$ 0000.
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19. The term "obverse" is borrowed from Morris 1998, 185, where it is used to organize six triadic transformations into three pairs (P/P', L/L', and R/R').
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20. To see why contrary motion is forbidden, consider starting with {0, 3, 6, B} and have two voices move in contrary motion. For example, if pc 0 moves to pc 2 and pc 6 moves to pc 4 a dead end is reached because there will be no way to reach pcs 8 and A via ic 2 motion without first returning to a previously left pc (0 or 6), which is forbidden. Another example, again starting with {0, 3, 6, B}: if pc 0 moves to pc 2 and pc B moves to 9, creating {2, 3, 6, 9}, one could then move pc 3 to 5 and 6 to 8, but with the B–9 and 3–5 voices moving in opposite directions, pc 1 becomes unreachable.
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21. The complete list of series: W_3–W_B–W_3–W_5, W_3–W_B–W_9–W_B, W_3–W_B–W_6–W_8, W_3–W_5–W_3–W_B, W_3–W_5–W_9–W_5, W_3–W_8–W_6–W_B, W_3–W_8–W_3–W_8; W_9–W_5–W_9–W_B, W_9–W_5–W_6–W_8, W_9–W_5–W_3–W_5, W_9–W_B–W_9–W_5, W_9–W_B–W_3–W_B, W_9–W_8–W_9–W_8, W_9–W_8–W_6–W_5; W_6–W_8–W_3–W_B, W_6–W_8–W_9–W_5, W_6–W_5–W_9–W_8, W_6–W_5–W_6–W_5, W_6–W_B–W_3–W_8, W_6–W_B–W_6–W_B, and all of their retrogrades. Such series can also be constructed in other MORRIS_w, such as MORRIS_1's $D^{ø7}$–D^7–$C^{ø\frac{6}{5}}$–$E♭^7$–$C♯^{ø\frac{6}{5}}$, which articulates W_4–W_6–W_7–W_6.
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22. The circle-of-fifths transformation, which amounts to pc multiplication by 5, "maps the chromatic scale onto the circle of fifths and vice versa" (Mead 1994, 36).
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23. Such VL exhibit "entirely uniform" voice leading, as defined by Straus 2003, 315.
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24. Two notes on this terminology. First, these "groups" are not, strictly speaking, mathematical groups. Second, notice that the third schritt group employs $-w$ in its subscripts whereas the third wechsel group employs $2w$. This apparent inconsistency is vital because it facilitates the discussion of similarities between transformations in different MORRIS+_w. For instance, W_{2w} refers simultaneously to four transformations—MORRIS+_1 W_2, MORRIS+_2 W_4, and MORRIS+_4 W_8, MORRIS+_5 W_A—which share important voice leading features with one another. For the same reason, subscripts including $-w$ are necessary in the third schritt group.
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25. The other matrix position sets are {(1, 4), (2, 3), (3, 2), (4, 1)}, {(1, 3), (2, 4), (3, 1), (4, 2)}, and {(1, 2), (2, 1), (3, 4), (4, 3)}.
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26. Category numbering is arbitrary; that is, category 1 in Example 13c does not correspond to category 1 in other examples.
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27. This is true because any pair of dyads has at least two sm interpretations and each sm interpretation has at least one VL that articulates it. For example, given the voice pair {D♭–B, C–D}, C and D♭ can be interpreted as sm 1 and sm 4, respectively, of a 4-13 in prime orientation, or as sm 4 and sm 1, respectively, of an inverted 4-13. Given the former interpretation, sm 1 (C) must move by ci 2 (to pc D) and sm 4 (D♭) must move by ci A (to pc B) and so a VL with oci 2 _ _ A is needed, either W_8 2A2A or W_B 22AA. Given the latter sm interpretation, sm 4 (C) must move by ci A (to pc D) and sm 1 (D♭) must move by ci 2 (to pc B) and so, once again a VL with oci 2 _ _ A is needed, either W_8 2A2A or W_B 22AA.
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28. The model's voice leading is strictly maintained throughout, except between the second and third piano chords of measure 3.
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29. The W_{2w} transformations of MORRIS+_2 and MORRIS+_4 have fewer than four in this chart because they are the special cases that result in uic $|x|^4$ mentioned above.
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30. Chord progressions in which two voices move by one pc interval and the other two voices move by another are addressed by "dual transformations" in O'Donnell 1997 & 1998.
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31. The reader may find it helpful to refer back to the T-matrices, which are provided in Example 12b.
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32. With all of the chords in prime orientation and the featured voice stating pc interval 5, S_1 **7**BA0, S_2 20B**7**, S_A B**7**A0, and S_5 B20**7** are of no use. Moreover, S_5 B2**0**7 is useless because no VL have **5** in matrix column 3, which leaves them unable to connect to S_5 B2**0**7, whose **0** is in matrix row 3.
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33. These are S_7 140B, S_7 131B, S_A 16AB, S_A 113B, W_7 1B1B, W_7 148B, W_A 11BB, and W_A 12AB.
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34. For example, any 4-12 in prime orientation can be realized as a dominant seventh chord with missing fifth and added minor ninth (C–E–B♭–D♭ ordered low to high) or as a minor triad with missing fifth and added major ninth and augmented eleventh (B♭–D♭–C–E). Another example, the set of voices {C–C, F–E, A♭–G, B–B♭} articulates 4-18 then 4-27, or, in tonal terms, V^7 of F minor with appoggiaturas that articulate an incomplete applied vii°^{7}, which is especially clearly related to conventional tonal practice if C–C appears in the bass voice.
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Prepared by Michael McClimon, Editorial Assistant