Building on Robert Morris's (1990) research on hexachordal ZC-relations, Stephen Soderberg (1998) identifies a constellation of ten hexachords that embed either one diminished seventh chord or two diminished triads. Soderberg divides the constellation, called MORRIS (or T-HEX), into four overlapping eight-hexachord sub-constellations based on tetrachordal subset content. The first of these sub-constellations, TRISTAN, includes the hexachords that embed two instances of set class 4-27$^{[0258]}$, the set class of the major-minor and half-diminished seventh chords. Similarly, constellations ZAUBER, AGITATION, and BROODING include the hexachords that embed two instances of set classes 4-18$^{[0147]}$, 4-13$^{[0136]}$, and 4-12$^{[0236]}$, respectively. Soderberg characterizes each of these tetrachordal set types as a “warp” of the diminished seventh chord. When the “warp index,” $w$, is 1, the result is set class 4-27—that is, moving any pc of a diminished seventh chord by interval class (ic) 1 creates a member of set class 4-27. Similarly, setting $w$ = 2, 4, and 5 creates set classes 4-18, 4-13, and 4-12, respectively. The article goes on to point out a general property of voice leading: in each hexachord the pair of tetrachords can be connected by holding two pitch classes in common and by moving two others by $\pm w$. The cases involving 4-27 and ic 1 voice leading (TRISTAN) are familiar—ii$^{[02]}$–V$^{[07]}$, the Tristan chord with resolution, A$^{[07]}$–B$^{[07]}$ at the beginning of Debussy's Faune, and others—and have been addressed in the theoretic literature by several authors. Example 1 presents the MORRIS constellation and its four overlapping sub-constellations, henceforth called...
MORRIS$1$, MORRIS$2$, MORRIS$4$, and MORRIS$5$, with each subscript indicating the warp index, $\nu$.

[2] Taking Soderberg’s MORRIS constellation as a starting point, this study explores a wide variety of voice-leading transformations involving set types 4-27, 4-18, 4-13, and 4-12. It starts with the tetrachordal voice-leading transformations that produce the MORRIS$\nu$ hexachords—the nine ways that a tetrachord may be connected to another in the same set class by holding two pitch classes in common and by moving two others by $\pm \nu$. (For each value of $\nu$ there are nine transformations but only eight hexachord types because two of the transformations yield the same hexachord type.) The article then greatly expands the scope of inquiry, not only by allowing a given tetrachord to connect to any member of the same set class, but also by considering all twenty-four ways to voice-lead each of these twenty-four tetrachordal connections. As a result, each of these much larger voice-leading spaces—MORRIS$+$1, MORRIS$+$2, MORRIS$+$4, and MORRIS$+$5—contains 576 ($= 24 \times 24$) voice-leading transformations. Each MORRIS$\nu$ is a subset of its corresponding MORRIS$+\nu$.

[3] The paper provides an organized view of the entire MORRIS$\nu$/MORRIS$+\nu$ system, but it also develops other voice-leading spaces within MORRIS$\nu$/MORRIS$+\nu$, each of which involves only a few of the MORRIS$\nu$/MORRIS$+\nu$ voice-leading transformations. The additional voice-leading spaces usually also include rules for requiring, preferring, allowing, or forbidding one transformation to follow another. Although the number of potential voice-leading spaces is infinite, my reasons for creating the ones I do are simple—to exercise melodic control over intervalic characteristics of the voices and to exercise harmonic control, not only of the individual tetrachords, but over the total pc content of sets of adjacent tetrachords. For example, in one space defined below a set of six MORRIS$5$ voice-leading transformations can be concatenated in any order but remain within a single octatonic collection, in another space four MORRIS$+4$ transformations are strung together so that one voice descends chromatically while the other voices also move stepwise, and in a third example ten MORRIS$+2$ transformations are arranged so that each voice is saturated with a different interval class.

[4] For an introduction to the variety exhibited by the voice-leading transformations within the MORRIS$+\nu$ universe, consider the musical excerpts in Example 2. Example 2a provides a transformation from MORRIS$2$, a pair of 4-18 tetrachords connected by ic-2 voice leading in two voices and by common tones in the other two, producing 6-30[013679]. An example from MORRIS$+1$, the excerpt from Debussy’s Faune in Example 2b includes a pair of 4-27 tetrachords (G$\#$, B$\#$) with the same ic in each voice (ic 1) and an overall octatonic collection. The excerpt from Berg’s Wozzeck in Example 2c provides a case from MORRIS$+2$, in which a pair of 4-18 tetrachords state ic 2 in two voices and ic 4 in the other two. Example 2d identifies a voice-leading transformation from MORRIS$+4$, a pair of 4-13 tetrachords with a different ic in each voice (ics 3, 0, 5 and 2). As shown in Example 2e, this four-voice model appears within the opening phrase of the vocal line in the third song of Berg’s Altenberg Lieder. Despite the linear realization, individual voices of the four-voice model are clear because each 4-13 articulates the same pitch contour 2–1–0–3. For example, ic 3 is clear in the “upper voice” because it is articulated by B$\#$4, the last and highest pitch of the first 4-13, and C$\#$5, the last and highest pitch of the second 4-13.

[5] The article builds on previous research in transformational theory and voice leading in several ways. First, following work of Julian Hook and others that deals with triads and seventh chords, the paper uses contextually-defined schritt ($S_n$) and wechsel ($W_n$) transformations to define mappings between tetrachords within the same set class. Second, in his article on voice leading with set type 4-27, Adrian Childs (1998) develops transformation labels that identify contrary/parallel motion in the moving voices as well as the harmonic interval classes formed by the sustained and moving voice pairs; I employ an adapted version of this labeling system to address all four tetrachord types. Third, branching out from the research on parsimonious voice leading—which focuses on stepwise melodic motion and tends to emphasize situations where moving voices articulate the same melodic interval class—this paper embraces both stepwise and non-stepwise voice leading and includes cases where voices move by the same or by different interval classes.

[6] Further, building on Robert Morris’s research in voice leading and compositional spaces, the paper utilizes T-matrices to investigate myriad voice-leading possibilities; develops several ways to measure voice leading, one of which is an unordered count of voice-leading interval classes that recalls Morris’s “voice-leading lists”; and suggests compositional applications of the theory. All voice leading in the article is one-to-one and onto, that is, within each tetrachord each pc appears in exactly
one voice and pcs are neither doubled nor omitted.\(^8\) Finally, as is customary throughout much of the literature, networks graphically depict abstract and literal voice-leading spaces.\(^9\)

7 The paper is in five parts. Part I offers preliminary definitions of the transformation and voice-leading concepts used throughout the remainder of the paper. Part II introduces the MORRIS\(_p\) voice-leading spaces. Part III suggests how to concatenate the voice-leading transformations within each MORRIS\(_p\) while being sensitive to recurring patterns of moving and holding within individual voices, aggregate completion, and related issues. Part IV introduces MORRIS+1, MORRIS+2, MORRIS+4, and MORRIS+5, categorizes their voice-leading transformations according to various properties so that a coherent view of the entire system emerges, suggests several compositional applications, and provides an analysis of an excerpt from Bartok's String Quartet No. 6. Part V shows how to put together lengthy strings of MORRIS+\(_p\) voice-leading transformations while maintaining control over the intervallic content of individual voices.

**Part I. Preliminary Definitions**

8 Part I defines several features of pitch-class sets (prime and inverted orientations, conventional ordering, set members) to prepare for a discussion of schritt and wechsel transformations (S\(_n\)\(\backslash\)W\(_n\)). Three types of intervals are then defined; two are familiar (pc interval, interval class) and one is new (contextual interval). This sets the stage for the definition of voice-leading transformations (VL), as well as for ordered and unordered lists of interval classes and contextual intervals. Part I concludes with discussions of T-matrices and inverse- and Z-related VL.

9 A set is said to be in prime orientation if it is related to its prime form by T\(_n\) and in inverted orientation if related to its prime form by T\(_n\)\(\backslash\)I. For example, within set class 4-18\(\{0,1,4,7,0\}\), \{1, 4, 7, 0\}, \{2, 5, 8, 1\}, and \{3, 6, 9, 2\} are in prime orientation because they are related by T\(_n\) to their set class prime form and \{7, 4, 1, 8\}, \{6, 3, 0, 7\}, and \{5, 2, B, 6\} are in inverted orientation because they relate to the prime form via T\(_n\)\(\backslash\)I. We will adopt the conventional ordering of pcs within tetrachords set instanced by the sets just mentioned. That is, for sets in prime orientation the first three pcs articulate an ascending root-position diminished triad (e.g. \{1, 4, 7, 0\}), and for sets in inverted orientation the first three pcs articulate a descending root-position diminished triad (e.g. \{7, 4, 1, 8\}). Based on this conventional ordering, individual pcs within a set are designated as set members (sm) 1, 2, 3, and 4, respectively. That is, in \{1, 4, 7, 0\}, pc 1 is set member (sm) 1, pc 4 is sm 2, pc 7 is sm 3, and pc 0 is sm 4. Within \{7, 4, 1, 8\}, pc 7 is sm 1, pc 4 is sm 2, pc 1 is sm 3, and pc 8 is sm 4.\(^10\)

10 A *schritt* set transformation, S\(_n\), is defined to articulate T\(_n\) when applied to a set in prime orientation, but T\(_n\)-1 when applied to a set in inverted orientation. For example, S\(_1\) transforms \{1, 4, 7, 0\} into \{2, 5, 8, 1\}, which articulates T\(_1\), and S\(_1\) transforms \{7, 4, 1, 8\} into \{6, 3, 0, 7\}, which articulates T\(_B\). S\(_0\), the identity transformation, transforms any set onto itself. Concerning *wechsel* transformations, when W\(_n\) transforms a set in prime orientation into an inverted one the embedded diminished triads articulate T\(_n\)-1 but when it transforms an inverted set into a prime one the embedded diminished triads articulate T\(_n\). For example, W\(_1\) transforms \{0, 3, 6, 1\} into \{7, 4, 1, 6\}, within which \{0, 3, 6\} and \{1, 4, 7\} articulate T\(_1\); W\(_1\) transforms \{6, 3, 0, 5\} into \{B, 2, 5, 0\}, in which \{0, 3, 6\} and \{B, 2, 5\} articulate T\(_B\). It would have been possible to define the W subscripts by chronicling the movement of any referential pc within the tetrachords. The use of the diminished triad strikes me as best because it allows a single rule for all four set types and has other advantages. For instance, it engages Straus’s 1997 notion of “near-transposition” (\(*T\_n\)\); that is, each W\(_n\) articulates \(*T\_n\) when applied to a prime set and \(*T\_n\)-1 when applied to an inverted set because it moves all but one of the set’s pcs by the same pc interval.\(^11\)

11 In order to compare features of the different MORRIS+\(_p\) voice-leading spaces, this paper often characterizes a set of transformations in terms of u, the warp index. For example, transformation S\(_8\) refers to MORRIS+1 S\(_1\), MORRIS+2 S\(_2\), MORRIS+4 S\(_4\), and MORRIS+5 S\(_5\). Similarly, W\(_{w+3}\) refers to MORRIS+1 W\(_4\), MORRIS+2 W\(_5\), MORRIS+4 W\(_7\), and MORRIS+5 W\(_8\).

12 This paper uses three types of voice-leading intervals. The first two types are familiar: given a voice in which pc x in tetrachordal set X is followed by pc y in tetrachordal set Y, the *pc interval from pc x to pc y is y=x mod 12 and the interval class (ic) between pcs x and y is \(|y−x|\ mod 12*). (Henceforth all arithmetic is considered to be mod 12 unless otherwise noted.) For example, the pc interval from 2 to 5 is 3, from 7 to 4 is 9, and from 1 to B is A, and the interval class between 2 and 5 is...
3, between 7 and 4 is 3, and between 1 and B is 2. Like the subscripts of the \(S_n\) and \(W_n\) transformations, the third type of interval depends on the (prime or inverted) orientation of the initial set. If set \(X\) is in prime orientation the contextual interval (ci) from \(x\) to \(y\) is \(y-x\) (the same as the pc interval), but if \(X\) is in inverted orientation then the contextual interval from \(x\) to \(y\) is \(x-y\) (the inverse of the pc interval). For example, if pc 2 occurs within a set in prime orientation, such as \(\{0, 3, 6, 2\}\), then the ci from pc 2 to pc 5 is 3; but if pc 2 appears within a set in inverted orientation, such as \(\{2, B, 8, 0\}\), then the ci from pc 2 to pc 5 is 9. (The orientation of set Y does not affect the ci.)

[13] Given any pair of tetrachordal sets X and Y, there are twenty-four ways to organize pcs into four voices so that each pc in each set appears precisely once. Therefore, there are twenty-four ways to voice-lead each \(S_n/\overline{W_n}\) transformation. The paper identifies each of these possibilities using ordered lists of contextual intervals (oci). For example, oci 8259 indicates that sm 1 of X moves by ci 8, that sm 2 of X moves by ci 2, that sm 3 of X moves by ci 5, and that sm 4 of X moves by ci 9. A voice-leading transformation, hereafter VL, can be minimally defined by tetrachord type and oci. [12] For example, 4-13[0136] and oci 8259 define a VL that can be articulated by the set of voices \(\{0–8, 3–5, 6–B, 1–A\}\). That is, sm 1 of X is pc 0, which moves by ci 8 to pc 8, sm 2 of X is 3, which moves by ci 2 to pc 5, sm 3 of X is 6, which moves by ci 5 to pc B, and sm 4 of X is 1, which moves by ci 9 to pc A. Throughout the paper, we will indicate tetrachord type with a voice-leading space (such as MORRIS+4 for 4-13) and include the pcset transformation that the VL articulates, creating VL labels such as MORRIS+4 \(W_5\) 8259. [13] Example 3a provides three realizations of this VL. The first two use the set of voices \(\{0–8, 3–5, 6–B, 1–A\}\) and the third \{5–9, 2–0, B–6, 4–7\}, which begins with a chord in inverted orientation. [14] As various VL are compared and contrasted, it is useful to provide other ways to characterize them. The unordered list of contextual intervals (uci) uses superscripts to count the number of times each ci appears within a VL; for instance MORRIS+4 \(W_5\) 8259 articulates uci 2 1 5 1 8 1 0 1. The unordered list of interval classes (uic) converts ci to ic; for example MORRIS+4 \(W_5\) 8259 yields uic 2 1 3 1 4 1 5 1. On a larger level, these uci and uic organize into types based on their superscripts. There are five uci types: \(x^1, x^2, x^3, x^4\), and \(x^5\), where \(x, y, z,\) and \(q\) are different ci. For example, uci 2 1 5 1 8 1 0 1 (and all others with one instance of four different ci) are of the type \(x^1 y^1 z^1 q^1\). There are also five uic types: \(|x|^1, |x| y^1, |x| y^2, |x| y^3, \) and \(|x| y^4\), where \(x, y,\) and \(q\) are different ic. For instance, uic 2 1 3 1 4 1 5 1 and all others with one instance each of four different ic are of the type \(|x|^1 y^1 z^1 q^1\). Concerning the relation of a VL’s uci and uic, when all ci in a uci articulate different ics (e.g. uci type \(x^1 y^1 z^1 q^1\) where \(x \neq y, x \neq z, x \neq q, y \neq z, y \neq q,\) and \(z \neq q\)) the relation between uci type and uic type is straightforward, as in the case of uci 2 1 5 1 8 1 0 1 and uic 2 1 3 1 4 1 5 1. But when different ci articulate the same ic (e.g. \(x = q\)), then the uci and uic superscripts will differ. For example, MORRIS+4 \(W_5\) 824A articulates uci 2 1 4 1 8 1 A 1 but uic 2 4 2 2.

[15] A T-matrix provides an efficient way to consider all VL that articulate a given transformation. Set \(X\) appears to the left of the matrix in conventional ordering and set \(Y\) appears above the matrix in conventional ordering. Matrix rows are labeled 1–4 from top to bottom to correspond to the set members of \(X\), and columns are labeled 1–4 from left to right to correspond to the sm of \(Y\). Matrix position \((j, k)\) contains the contextual interval from sm \(j\) of \(X\) to sm \(k\) of \(Y\). A one-to-one and onto mapping of \(X\) onto \(Y\) involves four matrix positions that include each matrix row and column precisely once. As an example, consider the matrix for MORRIS+4 \(W_5\), where \(X = \{0, 3, 6, 1\}\) and \(Y = \{B, 8, 5, A\}\), given as Example 3b. The boldface and underlined ci within the matrix point out VL MORRIS+4 \(W_5\) 8259—ci 8, 2, 5, 9 at matrix positions \((1, 2), (2, 3), (3, 1),\) and \((4, 4)\), respectively. [15] Notice that the notation MORRIS+4 \(W_5\) 8259 does not explicitly indicate the various sm of \(Y\), but that viewing the VL in its matrix context does make them explicit—the sm of \(Y\) are the column numbers in the matrix positions. For example, ci 8 at matrix position \((1, 2)\) means that sm 1 of \(X\) moves to sm 2 of \(Y\) via ci 8, ci 2 at matrix position \((2, 3)\) means that sm 2 of \(X\) moves to sm 3 of \(Y\), and so forth. Later in the paper, when the sm of \(Y\) is not vital to the discussion I use the compact notation (e.g. MORRIS+4 \(W_5\) 8259), but when the goal sm is vital—as in Part V when it comes time to control the intervallic content of a single voice in a lengthy chain of VL—I list ci along with their matrix position (e.g. ci 8 @ \((1, 2)\), etc.). Example 3c lists the twenty-four VL that articulate transformation MORRIS+4 \(W_5\), along with their uci and uic interpretations.

[16] Two VL are inverse if and only if applying one then the other results in an overall \(S_0\) 0000. Inverse-related wechsel VL articulate the same pcset transformation and the same uci, whereas inverse-related schritt VL articulate inverse pcset.
transformations and inverse sets of ci; for example, the inverse of MORRIS+4 \( W_5 8259 \) is MORRIS+4 \( W_5 5829 \) and the inverse of MORRIS+4 \( S_1 1186 \) is \( S_B \) BB64. Some VL are their own inverses, as with MORRIS+4 \( S_6 \) 0606 and MORRIS+4 \( W_4 \) 8286 (Example 3d). (16) VL are said to be Z-related if and only if they articulate the same pcset transformation and uic. Inverse-related wechsel VL are Z-related by definition, but there are often non-trivial Z relations. For instance, MORRIS+4 \( W_4 \)'s 87BA, A8B7, B247, and B724 each articulate uic 1214151, two pairs of inverse VL that create a Z-quadruple (Example 3e). (17)

Part II. VL within the MORRIS+ Voice-Leading Spaces

(17) Part II derives and lists the MORRIS+ VL, then introduces the Childs-derived labels, which help to compare and contrast various VL. Example 4 features four lattices that facilitate deriving the MORRIS+ VL—the nine ways to transform each MORRIS+ tetradchord into another member of the same set class by holding two pitch classes in common and by moving each of the other two pitch classes by interval class \( w \). Each lattice has \( <w> \)-cycle rows and \( <3> \)-cycle columns. Within each lattice, each row represents a voice, so that voice leading by ci \( w \) is modeled by horizontal movement to the right to an adjacent pc within a row and voice leading by ci \( -w \) by movement to the left. Vertical pc moves are forbidden. Having one pc in each row creates a tetrachord, which is a member of the MORRIS+ tetrachordal set class if and only if three pcs are in one column and one pc is in an adjacent column. Taking MORRIS+2 as an example, starting with \( \{0, 3, 6, 2\} \), there are twenty-four possible ways to move two voices by ic 5—six possible pairs of moving voices and two possible directions for each moving voice—but only nine of these create another 4-12. Since pc 2 is in a column by itself it is helpful to organize these nine possibilities around what pc 2 does. First, if pc 2 leads to pc 7 a dead end is reached because pc 7 would be two columns away from the other pcs and with only one move remaining it is impossible to move all three remaining voices to an adjacent column. If pc 2 leads to the left, to pc 9, there are six other single moves that generate another 4-12. Three of these involve moves to the right (0–5, 3–8, and 6–B) and the other three involve moves to the left (0–7, 3–A, and 6–I). Third and finally, if pc 2 does not move, then two of the other voices need to be led to the right to generate another 4-12. There are three ways to do this (0–5 with 3–8, 0–5 with 6–B, and 3–8 with 6–B). Overall, there are three cases where both moving voices articulate ci 5 (move to the right) and so the uci is \( 025 \), three cases where both moving voices articulate ci 7 (move to the left) and so the uci is \( 027 \), and three cases where one moving voice articulates ci 5 and the other ci 7 resulting in uci \( 02517 \). In the same way one could generate the other three sets of MORRIS+ transformations.

(18) Example 5 illustrates all of the resulting MORRIS+ VL in staff notation, along with their uci and uic interpretations. Each set of nine VL divide into three groups based on uci. In each case, \( S_3 \) \( u_00(-w) \), \( S_6 \) \( 0u0(-w) \), and \( S_9 \) \( 00w(-w) \) articulate uci \( 02w^{-1}(-w) \), \( W_3 \) \( (-w)00(-w) \), \( W_6 \) \( 0(-w)0(-w) \), and \( W_9 \) \( 00(-w)(-w) \) articulate uci \( 02^{-1}(-w) \); and \( W_{3+4} \) \( 0u00, W_{6+4} \) \( u000 \), and \( W_{9+4} \) \( 0u00 \) articulate uci \( 0w^{-2} \). In MORRIS+5 for example, \( S_3 \) \( 00507 \), \( S_6 \) \( 05070 \), and \( S_9 \) \( 05707 \) articulate uci \( 025171 \); \( W_3 \) \( 70057 \), \( W_6 \) \( 70707 \), and \( W_9 \) \( 07707 \) articulate uci \( 02722 \); and \( W_8 \) \( 0550, W_B \) \( 55050 \), and \( W_2 \) \( 5500 \) articulate uci \( 0252 \). These three groups of three correspond to the three general ladder-movement possibilities mentioned above: one move in each direction, two moves to the left, and two moves to the right, respectively. In each MORRIS+ set, all nine transformations articulate uci \( 02w^{-2} \). (18)

(19) The example also lists the hexachord types that are the focus of Soderberg’s article. Four set types appear in each MORRIS+. \( S_3 \) \( u_00(-w) \) and \( S_9 \) \( 00w(-w) \) state 6-27[013469], \( S_6 \) \( 0w0(-w) \) states 6-30[013679], and \( W_3 \) \( (-w)00(-w) \) and \( W_9 \) \( 00(-w)(-w) \) state 6-Z42[012369] and 6-Z29[023679]. Two hexachords appear in only two constellations: \( W_8 \) \( 0(-w)0(-w) \) generates 6-Z28[013569] in MORRIS+1 and MORRIS+5 and 6-Z45[023469] in MORRIS+2 and MORRIS+4.

(20) The example also provides Childs-derived labels, in which each digit indicates an interval class, “h” stands for held, “c” for contrary motion and “p” for parallel. For example, the label for MORRIS+4 \( S_4 \) 4008 is h3c15, which indicates that the held pcs (3 and 6) articulate ic 3, the moving voices (0–4 and 1–9) progress in contrary motion, and create what I will call the moving dyads \( \{0, 1\} \) and \( \{4, 9\} \), which articulate ic 1 then ic 5. Such labels highlight five features of the VL. First, ics 3 and 6 play a salient role: each VL features ic 3 or 6, either as its held dyad or as its parallel moving dyads. Second, in MORRIS+1 and MORRIS+5, if a held dyad articulates an odd ic the moving dyads articulate even ics, and if the held dyad is even the moving dyads are odd (e.g. h3c24, h1p66, h6c11, h4p33), but in MORRIS+2 and MORRIS+4 the held and moving dyads within the
same VL are either all odd or all even (e.g. h3c15, h3p11, h6c22, h4p66). Third, within each MORRIS_w, even though S_6 0w0(w) and W_6 0(w)0(w) are different VL—one involves contrary motion and the other parallel—they involve the same held ic and the same moving dyad ics, as with MORRIS_w’s S_6 0507 = hgc111 and W_6 0707 = hgp11.

[21] Fourth, each Childs-derived label that includes ic 3 appears in two MORRIS_w systems; that is, VL that articulate S_3, S_0, W_3, W_0, W_+3+, and W_0+3+ in MORRIS_w have the same set of held and moving dyads as the corresponding transformation in MORRIS_w. For example, MORRIS_1 S_3 100B and MORRIS_5 S_3 5007 both articulate h3c24; MORRIS_1 W_3 000B and MORRIS_5 W_3 7007 both articulate h3p22; and MORRIS_1 W_3 0110 and MORRIS_5 W_8 0550—which are W_3+3 0w0 in both cases—articulate h2p33. Fifth and finally, W_3 and W_3+3+ VL swap their held and moving ics so do W_6 and W_6+3+, and W_9 and W_9+3+, creating what we will call obverse VL pairs. For example, MORRIS_2 W_9 00AA = h3p55 and W_9 2200 = h5p33, are obverses. Any VL followed by its obverse results in an overall S_w wwwww or S_−w (w)(w)(w)(w)(w). MORRIS_2 W_B 2200 then W_9 00AA articulates an overall S_2 2222, while MORRIS_2 W_9 00AA then W_9 2200 an overall S_A AAAA. Three other obverse pairs are shown in Example 6.

**Part III. Four Applications of MORRIS_w VL**

[22] There are innumerable ways to combine VL within a given MORRIS_w. As an introduction to this topic, part III explores four situations: it creates series of MORRIS_1 VL in which one voice is held throughout, concatenates MORRIS_5 VL in an octatonic context, and generates twelve-tone structures from MORRIS_2 and MORRIS_4 VL. Since long strings of VL will be considered and since each pcset transformation within the MORRIS_w universe is associated with only one VL, I will employ a short hand notation where a pcset transformation label stands for its corresponding VL; for example MORRIS_1 W_4 stands for MORRIS_1 W_4 0110, MORRIS_1 S_3 stands for MORRIS_1 S_3 100B, and so forth.

[23] One way to limit the possibilities for VL concatenation is to control the motion of one or more particular voices. For example, it is possible to arrange VL so that a particular voice is always held. As an example, the network in Example 7a suggests how to assemble MORRIS_1 VL so that pc 2 is held throughout. Network nodes identify the eight members of set class 4-27 that include pc 2 and lines connecting the nodes show which chords may be connected by MORRIS_1 VL. Each line is bidirectional, so that, for instance, D_7 may lead to D5^7 and vice versa. In most cases, both directions articulate the same transformation, as with W_4 which transforms D_7 into D5^7 and vice versa; but in four cases, one direction articulates S_3 and the other its inverse, S_0 (e.g. S_3 transforms G_7 into E_7 and S_0 transforms E_7 into G_7). Since D_7 and E5^7 each connect to three other chords, and the other six chords each connect to five others, there are still a huge number of possible paths through this network.

[24] To reduce the possibilities further we can stipulate that the path begins and ends at D_7 and includes each other chord precisely once, and that the voice beginning with pc 0 alternates holding and moving. Since each path begins with D_7, and by holding pc 0, the only other chord that includes pc 0, D5^7, must be the second chord. Pc 0 must then move by ic 1. Since no chords on the network include C^#, the next chord must be one of the four that includes pc B (B5^7, E_7, G_7, G^#5^7). Pc B then holds, and so the next chord must also be from this group. Since each chord in this group connects to two others in the group there are always two possibilities. Pc B must then move by ic 1. It may not move back to pc 0 (because there are no new chords that include pc 0) so it must move to pc A, which is harmonized by the only two chords that include it B5^7-E5^7 (or vice versa). In similar fashion the path returns to D_7 via the two chords that have not yet been included. All in all, there are ten possible paths, given in Example 7b. Two paths contain only wechsel VL and eight paths contain a mixture of wechsel and schritt VL. Example 7c realizes two of these paths in staff notation. In each case the held pc 2 appears in the bass voice and the pc-0 voice in the soprano.

[25] Another way to restrict choice is to limit the possible VL to fewer than nine; say, for example, the six that produce hexachords that are subsets of the octatonic collection. In MORRIS_5 for instance, these are S_3, S_0, W_8, W_9, W_B, and W_2. Any string involving these VL stays within the same octatonic collection. The polyrhythmic texture in Example 8 provides one such series, W_8–S_0–W_8–S_3–W_2–S_9–W_2–S_9–W_8–W_2, which uses only four of the six octatonic VL but which includes all eight instances of 4-13 within one octatonic collection. In this and all such octatonic series each voice alternates pc intervals w and w and therefore oscillates between two pitch classes; this occurs because consecutive w (or consecutive
would lead outside the octatonic collection.

[26] It is also possible to concatenate VL so that they create twelve-tone structures. For example, consider series of MORRIS_2 VL that embrace all twelve pcs as quickly as possible, that is, four pcs in the first chord and two new pcs in each of the next four. Put another way, each series uses each pc exactly once—a pc may be held from one chord to the next but once left it is not reiterated. Filling these conditions is only possible when all four voices move in the same direction and so schritt VL, which involve contrary motion, are forbidden. Since a series of wechsel VL creates an alternation between sets in prime orientation and those in inverted orientation, there must also be an alternation of contextual intervals 2 and A in order to keep the voices moving by the same pc interval. This means that if the first transformation is W_3, W_6, or W_9, which articulate uci 0 < A^2, then the next transformation must be W_5, W_8, or W_B, which articulate uci 0 > 2 (A^2). For example, transforming \{0, 3, 6, B\}, a set in prime orientation, by W_3 creates motion by pc interval A (0–A and B–9), which leads to \{A, 3, 6, 9\}, a set in inverted orientation. To continue motion by pc interval A, we need to choose W_5, W_8, or W_B; choosing W_B, for instance, creates moving voices 6–4 and 9–7, which lead to \{A, 3, 4, 7\}. At this point there are three possible continuations that complete the aggregate: W_3–W_5, W_9–W_B, and W_6–W_8.

[27] Musical realizations of these three complete series, W_3–W_5–W_9–W_B, and W_3–W_B–W_6–W_8, appear in the first system of Example 9. The second system presents the other four series that begin with W_3. There also exist seven series that begin with W_9 and six that begin with W_B. All twenty of these series feature the same set of four voices (0–A–8, 6–4–2, 3–1, and B–9–7–5) but with varying pc repetitions. For instance, in various series the 0–A–8 voice is articulated as 0–A–A–A–8, 0–A–A–8–8, 0–0–A–8–8, 0–0–A–A–8, and 0–0–0–A–8. Finally, all twenty of these series may also be played in retrograde, for a total of forty MORRIS_2 series that complete the aggregate as quickly as possible. (21)

[28] MORRIS_4 VL feature set type 4-13[0136] and voice leading by ic 4. Since 4-13 creates the pc aggregate when transposed by 4 and 8, these VL are ideally suited to produce twelve-tone designs. Consider the matrix in Example 10a, whose rows and columns are saturated with MORRIS_4 VL, each expressed as a series of three dyads. For example, \{0, 1\}–\{3, 6\}–\{8, 9\} in the top row expresses W_3 = h3p11. The held pcs, 3 and 6, appear in the middle dyad, surrounded by the moving dyads, so that both 4-13 are clear (\{0, 3, 6, 1\} and \{3, 6, 9, 8\}). The moving voices, 0–8 and 1–9, are articulated by pcs in the outer dyads. Completing the top row, \{8, 9\}–\{B, 2\}–\{4, 5\} and \{4, 5\}–\{7, A\}–\{0, 1\} replicate this W_3 transformation at T_4 and T_8, creating a twelve-tone cycle that wraps around to its starting point. Offset by one dyad, this row also thrice embeds W_7 = h1p33, the obverse of W_3 = h3p11: \{3, 6\}–\{8, 9\}–\{B, 2\} and its T_4 and T_8 transformations. The second-highest row is a retrograde rotated circle-of-fifths transformation of the top row and therefore embeds W_0 = h3p55 and W_1 = h5p33. The remaining rows and columns are T_0/T_4/T_8 transformations of these two. (22)

[29] Example 10b realizes the matrix for three pianists, one staff/hand/register for each matrix row and one quarter-note beat for each column. Instead of realizing the entire matrix at once, measure 1 articulates the upper-right portion of the matrix (including the main diagonal) and measure 2 the lower left (also including the main diagonal), so that each staff begins and ends with the dyad from the main diagonal, as with the top row’s \{0, 1\}, the second row’s \{0, 5\}, and so forth. As a result, the full columnar aggregate appears only twice, at the end of measure 1 and the beginning of measure 2.

Part IV. MORRIS+_a: Other Voice-Leading Possibilities

[30] Part IV considers four larger sets of possibilities involving the same four tetrachord types, each of which embraces all twenty-four ways to transform a given set into another member of the same set class and all twenty-four VL for each transformation. These greatly expanded spaces are labeled MORRIS_+1, MORRIS_+2, MORRIS_+4, and MORRIS_+5. The discussion first introduces the additional schritt and wechsel transformations (with one VL for each) and then addresses the multiple VL possibilities.

[31] Example 11a presents W_w, W_0, and W_2w within each MORRIS+_a. W_w features a VL in which the pcs of the diminished triad move by ic w and the other pc by −w, so that all four pcs move by ic w (that is uci = w^4). See, for example, MORRIS_+2 W_2 222A (uci 2^4). The total pc content of each W_w transformation is an octatonic collection. Each W_0 features a VL that holds the diminished triad invariant and moves the other voice by ic −2w; for instance, MORRIS_+5 W_0
0062 sustains the pcs of the diminished triad while the other pc articulates ci 2. Conversely, each \( W_{2w} \) features a VL that moves the diminished triad by ci 2\( w \) and sustains the other pc, as with MORRIS+5 \( W_A \) AAA0, which moves the pcs of the diminished triad by ci A while the other pc is sustained. The total pc contents of \( W_0 \) and of \( W_{2w} \) are abstract complements of one another, as in MORRIS+5, where they articulate 5-8[02346] and 7-8[0234568], respectively. Example 11b presents the remaining trio of wechsel transformations within each MORRIS+: \( W_{3+2w}, W_{6+2w}, \) and \( W_{9+2w} \) (e.g. MORRIS+4 \( W_B, W_2, \) and \( W_3 \)). Note that each MORRIS+ connects to each of the three others via transformations that share the same total octachordal pc content. For example, MORRIS+1’s \( W_5, W_8, \) and \( W_B \) share 8-23[0123578A], 8-24[0124568A], and 8-20[01245789] with MORRIS+4 \( W_5, \) MORRIS+5 \( W_4, \) and MORRIS+2 \( W_1, \) respectively.

[32] The remaining nine transformations within each MORRIS+ are the schritt transformations other than \( S_3, S_6, \) and \( S_9, \) which are easy to envision: transpositionally-related chords in which all voices move in parallel motion by ci n. (23) (\( S_0 \) is the identity transformation, which transforms a set onto itself.) These schritt transformations articulate a few interesting features concerning total pc content. First, \( S_w \) and \( S_{-w}, \) the only schritt transformations with octachordal total pc content, generate the complement of the MORRIS+2 \( |2w| \) tetrachord. When \( w = 1 \) or 5, \( |2w| = 2, \) which means that MORRIS+1’s \( S_1 \) and \( S_6 \) and MORRIS+5’s \( S_5 \) and \( S_7 \) each create 8-18[01235689], the complement of MORRIS+2’s 4-18. Similarly, when \( w = 2 \) or 4, \( |2w| = 4, \) so that MORRIS+2’s \( S_2 \) and \( S_{-2} \) and \( S_A \) and \( S_{-4} \) generate 8-13[01234679], the complement of the MORRIS+2’s 4-13. The latter case is special because pairs of 4-13 tetrachords are generating their own complement. Second, for \( w = 1, 2, \) and 5—but not 4 because of the special case just mentioned—\( S_{2w} \) and \( S_{-2w} \) generate the same total septachordal pc content as \( W_2w. \) For example, MORRIS+1’s \( S_2, S_5, \) and \( W_2 \) each generate 7-34[013468A]. All in all, it will be helpful to divide the twenty-four transformations within each MORRIS+ into six groups as follows: The \( S_0 \)-group includes \( S_0, S_3, S_6, \) and \( S_9; \) the \( S_w \)-group includes \( S_w, S_{3+w}, S_{6+w}, S_{9+w}; \) and the \( S_{-w} \)-group includes \( S_{-w}, S_{3-w}, S_{6-w}, \) and \( S_{9-w}. \) The \( W_0 \)-group includes \( W_0, W_3, W_6, \) and \( W_9, \) the \( W_w \)-group includes \( W_w, W_{3+w}, W_{6+w}, \) and \( W_{9+w}, \) and the \( W_{2w} \)-group \( W_{2w}, W_{3+2w}, W_{6+2w}, \) and \( W_{9+2w}. \) (24)

[33] With all of the \( S_n/W_n \) transformations in place we now consider multiple VL possibilities for each, which, as outlined in part I, can be derived from T-matrices. Example 12a provides generalized matrices in terms of n and w (for \( w = 1, 2, 4, \) or 5), one for \( S_n \) and one for \( W_n, \) which allow general properties of the matrices and resulting VL to emerge. For example, each \( S_n \) matrix features ten different ci, four that appear multiple times and six others that appear only once. Each \( W_n \) matrix features eight different ci, one that appears once, one that appears twice, and six others that appear twice each. Each \( W_n \) matrix is symmetrical around the main diagonal. Many consistent patterns may be drawn from the matrices but I will mention only three. First, in every \( W_n \) matrix there are four sets of matrix positions that yield an oci with two instances each of two different ci (uci type \( x^2_t \). For instance, with the matrix for MORRIS+4 \( W_n \) the set of positions \{1, 1\}, \{2, 4\}, \{3, 3\}, \{4, 2\} yields oci xyxy, with uci \( x^2_t \) where \( x = 6+n \) and \( y = 6+n-w. \) (25) Second, the upper-left \( 3 \times 3 \) square of a \( W_n \) matrix depends on n but not \( w, \) which means that all four \( W_1 \) matrices have an identical upper-left \( 3 \times 3 \) square. Third, transformations in the same group feature the same set of ci in these \( w \)-independent matrix positions, a set equal to their subscripts taken as a set. For example, the \( W_0 \)-groups have ci 0, 3, 6, and 9 in their upper-left \( 3 \times 3 \) squares. These features can be verified by consulting Example 12b, which lists all ninety-six matrices.

[34] I will now organize the voice-leading possibilities within this expanded universe by uci type. VL articulate one of five uci types, \( |x|^4, |x^3||y|^1, |x^2||y|^2, |x^2||y|^2||z|^1, \) and \( |x^1||y|^1||z|^1|q|^1, \) all but the fourth of which are addressed below. The discussion of each uci type is in two parts. The first addresses the uci types that generate the uci type and the transformations and VL that create each uci type. The second identifies the possible uci within the uci type, lists the VL within each uci and groups the various uci into categories based on the presence/absence of ics 0, 3, and/or 6. Different uci within the same category contain the same (or similar) numbers of VL, which articulate the same (or similar) number and type of schritt/wechsel transformations. The discussion outlines the organization of the possible VL and suggests several compositional applications.

[35] The MORRIS+ system includes 2,304 VL. Of these, eighty articulate uci type \(|x|^4, 136 \) articulate uci type \(|x^3||y|^1, 432 \) uci type \(|x^2||y|^2, 904 \) uci type \(|x^2||y|^1||z|^1, \) and 752 uci type \(|x^1||y|^1||z|^1|q|^1, \)
The eighty VL that articulate uic type $|x|^4$ arise from three uci types: $x^4$, $x^3y^1$ where $x = -y$, and uci $x^2y^2$ where $x = -y$. Forty-eight VL articulate uci type $x^4$; these are the straightforward cases involving parallel voice leading. Eight VL arise from uci type $x^3y^1$ where $x = -y$: one VL from $W_n$ and one from $W_{n+2}$ within each MORRIS-+ ($e.g.$ MORRIS-+’s $W_1$ 111B and $W_7$ 7775). The other twenty-four VL have uci type $x^2y^2$ where $y = -x$. Each MORRIS-+ includes $S_3$ 9393 and $S_9$ 3939, a total of eight VL with uci $3^2$ and uic $3^4$. (The oci is identical for each MORRIS-+ because it involves array properties (3,1), (2,2), (1,3) and (4,4), whose values do not depend on the pc that differentiates the tetrachords from one another.) The remaining sixteen VL occur within the $W_{2n}$-groups of MORRIS-+ and MORRIS-+. In MORRIS-+, $W_4$ contains $B_11B$, 4848, and 7755, which produce uic $1^4$, $4^4$, and $5^4$, respectively. $W_4$ also articulates three such VL, and $W_7$ and $W_1$ one each. In MORRIS-+ it is $W_8$ and $W_5$ that include three each and $W_8$ and $W_2$ that have one. Example 13a lists all of these VL and Example 13b provides realizations of some of them in staff notation.

Example 13c divides the seven uic $(0^4, 1^4, 2^4, 3^4, 4^4, 5^4, 6^4)$ into three categories. The first category includes uic $0^4$ and $6^4$, each of which includes only four VL. The second category includes only uic $3^4$, which includes sixteen VL, the $Z$-pairs $S_3$ 3333/9393 and $S_9$ 9999/3939 in each MORRIS-+. Category 3 uic $1^4$, $2^4$, $4^4$, and $5^4$ each contain fourteen VL. In uic $1^4$ and $5^4$ these are distributed $3+5+3+3$ across the four MORRIS-+ spaces and in uic $2^4$ and $4^4$, $2+4+6+2$.

Example 13d provides a seven-chord, six-voice model created from the MORRIS-+ VL within uic $2^4$, with the top four voices articulating one string of VL and the bottom four voices another. Such models are easy to generate because any two voices saturated with ic 2 that do not form harmonic ic 4 (the ic that 4-13 does not contain) can participate in multiple statements of these VL. This model demonstrates variety in several respects. It incorporates all six MORRIS-+ VL with uic $2^4$ $(S_2$ 2222, $S_4$ AAAA, $W_5$ A22A, $W_7$ 2A2A, $W_{11}$ A2A2, and $W_{13}$ B22A) and mixes prime and inverted sets in various ways. At times it unfolds the same VL in both layers simultaneously while at other times the VL differ. The voice leading from one chord to the next usually mixes pc intervals 2 and 4, but the third and fourth chords articulate inverse-related VL, $S_2$ 2222 and $S_4$ AAAA, which, since they are applied to sets of contrasting orientation, create pc interval 2 in all voices. Moreover, the seven chords articulate six different set types; each of the first six chords articulates a MORRIS-constellation hexachord because it is comprised of two 4-13s that share two pcs, and the final chord has only four pcs—the same 4-13 in both layers. Example 13e provides a musical realization based on the model: the middle two voices of the model appear in the cello part, a mixture of double stops and compound melody, and the other model voices appear in the piano part, all in the treble register. The middle of the excerpt articulates the complete model, while the beginning and end offer fragments.

Example 14c organizes all 136 VL with uic type $|x|^3|y|^1$ by uic. There are four categories. Category 1 includes the four uic for which $y = 3x$: these are $3^1$, $3^2$, 3401, and 5331. Each of these four uic includes twelve $S_n$-based VL along with two or four $W_n$-based VL. The remaining three categories include only wechsel-derived VL. Each uic in category 2 includes eight VL, each one in category 3 twelve VL, and each one in category 4 only two.
[41] The 432 VL with uic type $|x|^2|y|^2$ articulate one of three uic types: uci type $x^2y^2$ where $x \neq -y$ (216 VL), uci type $x^2y^2z^2$ where $z = -y$ (136 VL), and uci type $x^2y^2z^2$ where $x = -z$ and $y = -q$ (80 VL).

[42] Of the 216 VL that articulate uci type $x^2y^2$ where $x \neq -y$, forty are $S_n(6+n)n(6+n)n$ where $n \neq 3$ or 9—$S_0$ 6060*, $S_1$ 7171, $S_2$ 8282, $S_4$ A4A4, and so forth. The remaining ones arise from $W_n$ transformations. Most MORRIS$_+$ $W_n$ articulate four such VL, one for each of the two dyad partitions that involves [03] and two for the partition that involves [06]. For example, MORRIS$_+$ $W_4$ features 8118, 7722, A5A5, and 4545, which articulate uci 1$^4$2$^2$, 2$^2$7$^2$, 5$^2$A$^2$, and 4$^2$5$^2$, and uic 1$^2$4$^2$, 2$^2$5$^2$, 2$^2$5$^2$, and 4$^2$5$^2$, respectively. In 8118, the embedded [03] dyad formed by sm 2 and sm 3 moves by ci 1 and the [02] dyad formed by sm 1 and sm 4 moves by ci 8; and in 7722, the [03] dyad formed by sm 1 and sm 2 moves by ci 7 and the [04] dyad (sm 3 / sm 4) moves by ci 2; in A5A5 and 4545 the [01] dyad (sm 2 / sm 4) moves by ci 5 and the [06] dyad (sm 1 / sm 3) moves by ci A in one case and by ci 4 in the other. The upper portion of Example 15a lists all of these VL. In Example 15b the VL mentioned in the discussion are realized so that the top two voices move in parallel motion by one ci and the bottom two voices move in parallel motion by another.[30]

[43] The 136 VL with uci type $x^2y^2z^2$ where $z = -y$ arise as follows: each MORRIS$_+$'s $S_0$ and $S_6$ include six such VL, each $S_3$ and $S_9$ includes four, each $W_{x_3}$ and $W_{x_9}$ three, and each $W_{y_3}$, $W_{y_9}$, $W_{y_3+y_3}$ and $W_{y_9+y_9}$ two. Each $S_0$ includes five VL that hold two pcs (ci 0) while the other two voices articulate a voice exchange ($) and one VL with a double voice exchange ($**$) (e.g. MORRIS$_+$ $W_2$ $S_0$ 0057*, 3900*, 0804*, B001*, 0390*, and 6864**), each $S_6$ includes one VL with a pair of ci 0 and five with a pair of ci 6 (e.g. MORRIS$_+$ $S_2$ 020A, 5667, 66B1, 9366, 6936, 626A*), and so forth. The middle part of the chart in Example 15a provides the complete list and Example 15c realizes these and other sample VL in musical notation.

[44] The 80 VL with uci type $x^2y^2z^2$ where $z = -x$ and $q = -y$ arise only from the $S_0$ group of each MORRIS$_+$ and from the $W_{2x}$ group of MORRIS$_+$ and MORRIS$_+$. Each $S_0$, $S_3$, $S_6$, and $S_9$ includes two VL, both of which involve ic 3 (e.g. MORRIS$_+$ $S_3$ 3984** and 239A**, $S_3$ 5397* and 932A and 8934, $S_6$ 932A and 8934, $S_9$ 3957*(and 9391)). Each $W_{2x}$-group transformation includes six VL organized into three $Z$-pairs; for instance, MORRIS$_+$ $W_4$ includes 715B/B715 (uic = 1$^2$4$^2$), B148/481B (uic = 1$^2$4$^2$), and 4758/7845 (uic = 4$^2$5$^2$). The lower part of Example 15a includes the complete list and Example 15c lists most of the cases just mentioned in staff notation.

[45] Examples 15e and 15f illustrate a compositional application involving one of these $Z$-pairs: MORRIS$_+$ $W_4$ 715B/B715. Example 15e includes four two-chord, four-voice models, labeled X 715B, Y 715B, X B715, and Y B715. X and Y denote tetrachord pairs ($X = \{0, 3, 6, B\}$–(A, 7, 4, B) and $Y = \{1, A, 7, 2\}$–(3, 6, 9, 2)) and 715B and B715 signify the motion of the voices. Each model is arranged so that pc interval B appears in the soprano register, pc interval I in the alto, pc interval 5 in the tenor, and pc interval 7 in the bass. Example 15f provides a short musical excerpt for piano that includes five different combinations of these models, in which voices sometimes appear an octave higher or lower than in the model in order to achieve more sonorous spacing and/or to avoid registral overlap. The excerpt first explores the four ways to combine either X 715B or X B715 with either Y 715B or Y B715. Although each combination results in the same pair of octachordal pesets and the same melodic motion within each register, there is considerable variety because each combination creates a different pair of parallel dyads within each register, as with the soprano's $\{B5, G6\}$–$\{A\#, F\#\}$, $\{B5, B\#6\}$–$\{A\#, A6\}$, $\{C7, G7\}$–$\{B6, F\#7\}$, and $\{C6, B\#6\}$–$\{B5, A6\}$, which articulate harmonic ics 4, 1, 5, and 2, respectively. The excerpt ends with the simultaneous unfolding of X 715B, X 715B, Y 715B, and Y B715, sixteen-voice harmony in which each pc of X and Y appears in two registers. Each register contains parallel tetrachords, 4-4[0125] in soprano and alto and 4-14[0237] in bass and tenor.

[46] Example 15g lists all of the VL of uic type $|x|^2|y|^2$, organized by uic. The twenty-one possible uic organize into six categories. Category 1 involves only ics 0, 3, and/or 6, that is, uic 0$^2$3$^2$, 0$^2$6$^2$, and 3$^2$6$^2$. All VL within these uic arise from $S_0$-group transformations and have identical uic in each MORRIS$_+$. Category 2 involves ics 0 but not ics 3 and 6. Each uic 0$^2$n$^2$ (0$^2$1$^2$, 0$^2$2$^2$, 0$^2$4$^2$, 0$^2$5$^2$) includes fourteen VL: nine VL that constitute the MORRIS$_n$ VL discussed at length in parts II and III of the paper, one $S_0$ VL in each of the other three spaces, and a further pair in MORRIS$_+$ 6-n. Category 3 involves ics 6 but not ics 0 and 3. Mirroring category 2, each uic 6$^2$n$^2$ (6$^2$1$^2$, 6$^2$2$^2$, 6$^2$4$^2$, and 6$^2$5$^2$) embeds fourteen VL: nine within MORRIS$_+$ 6-n, one $S_6$ VL in each of the other three spaces, and a further pair in MORRIS$_+$. 
The fourth category involves ic 3 but not ics 0 and 6. Each uic $3^2n^2 \ (3^21^2, 3^22^2, 3^24^2, \text{and } 3^25^2)$ embeds thirty-two VL, ten within MORRIS$_n$, ten within MORRIS$_{\text{m}-n}$, and six in each of the other two spaces. A fascinating subset of each uic in this category is the presence of four VL pairs that share not only melodic ics but harmonic ones. Consider as an example MORRIS$^+2$ W$_2$ 9B89 and MORRIS$^+4$ W$_4$ 9119. In each case, sm 2 and sm 3 create ic 3 harmonically and move by ic 1 melodically, and sm 1 and sm 4 create ic 1 harmonically and move by ic 3 melodically. Example 15h realizes this pair and the three other pairs within uic 1 232.

Categories 5 and 6 do not involve ics 0, 3, and 6. In category 5, one ic is odd and the other even (1 222, 1 242, 2 252, and 4 252) and in category 6 both are odd or both are even (1 252 and 2 242). Within category 5, each uic includes twenty VL, all of which articulate wechsel transformations. Within category 6, each uic consists of thirty-six VL, including a pair of Z-quadruples (e.g. MORRIS$^+2$ W$_1$ 4A28/84A2/4422/8AA8 and W$_7$ A482/2A48/2442/AA88 within uic 2 242).

The discussion of uic type $|x|^2 |y|^2$ concludes by pointing out a few such VL within an excerpt from Bartok's String Quartet No. 6. The excerpt in Example 16 is the opening of the piece, in which the outer voices present overlapping statements of the opening 3/2-measure melody (bracketed on the score) and then diverging sequences based on fragments of that melody. The inner voices are also primarily stepwise, moving first in free counterpoint and then in parallel tritones. Harmonically, the excerpt features five triadic anchor points, nineteen instances of the MORRIS tetrachords, and a variety of other sonorities. The MORRIS tetrachords create six VL of uic type $|x|^2 |y|^2$: two from MORRIS$_1$ (W$_9$ 008B and W$_7$ 1010), one from MORRIS$_2$ (S$_3$ 200A), a side-by-side pair from MORRIS$^+4$ (S$_B$ 5B5B and W$_A$ A0A0), and one from MORRIS$^+5$ (S$_0$ 6B01**). Two of these VL articulate uic 0 212, two uic 0 222 and the others 1 252 and 2 262. Four involve ic 0, which is not surprising given the frequency with which two voices hold while two others move, and all six involve either ic 1 or ic 2, which makes sense given the stepwise thematic material. In all but two cases “passing sonorities” complicate the musical realizations. The rhythmic offset of the moving voices of MORRIS$_2$ S$_3$ 200A creates an intervening 4-17[0347], parallel tritones moving by half step decorate W$_7$ 1010 with a pair of 4-16s [0157], and further stepwise motion during MORRIS$^+4$ S$_B$ 5B5B and MORRIS$^+5$ S$_0$ 6B01** creates embellishing 4-5 [0126] and 4-6 [0127] sonorities. Moreover, each VL features precisely two voices moving stepwise (either by ic 1 or ic 2), which creates contrast with the approaches to the F$\#$, GM, E$\#$M and Cm triadic anchors, each of which involves stepwise motion in all four voices. The approaches to F$\#$M and GM mix half- and whole-step voice leading and the approaches to E$\#$M and Cm feature all four voices moving by half step.) Finally, taken as a group, the roots of all five triadic anchors (A, F$\#$, G, E$\#$, C) articulate 5-31 [01369], the sole pentachordal set type that embeds all four of the MORRIS tetrachords.

There are 752 VL that articulate uic type $|x|^1 |y|^1 |z|^1 |q|^1$, all of which derive from uci type $|x|^1 |y|^1 |z|^1 |q|^1$ where $x \neq -y, x \neq -z, x \neq -q, y \neq -z, y \neq -q, \text{and } z \neq -q$. Each S$_n$ and W$_n$ generates between 20 and 48 VL of this uic type (consult Example 17a) and the VL organize into nineteen uic (which divide into six categories) as shown in Example 17b. The reader may wish to step through these charts on their own as I have done with similar charts above, or simply proceed with the discussion here, which provides a compositional application involving one sample uic and then highlights another special uic.

VL with uic type $|x|^1 |y|^1 |z|^1 |q|^1$ can be used to differentiate voices from one another. As an example, consider the twelve MORRIS$^+4$ VL with uic 0 123151: S$_0$ 0372/0A95, S$_3$ 90A5, S$_0$ 0732, W$_0$ 0295/5902, W$_3$ 9025/9502, W$_A$ 37A0/A073, and W$_1$ 73A0/7A03. Example 17c realizes these in staff notation so that ic 0 always appears in the soprano voice, ic 2 in the alto, ic 3 in the tenor, and ic 5 in the bass. Example 17d constructs a short excerpt for string quartet that incorporates three 0 123151 VL. The passage's 6-4 13 chords articulate S$_0$ 0372-(S$_0$ 3669)-W$_0$ 0295-(W$_7$ 777B)-S$_0$ 0372. For each 0 123151 VL, ic 0 appears in violin I, ic 2 in violin II, ic 3 in viola, and ic 5 in the cello. Since every second VL is part of 0 123151, each pc in the passage participates in a statement of its instrument's primary ic; for instance, violin II's D$\#$-E$\#$, A–B, and C–D state ic 2. The parenthesized VL, which do not articulate uic 0 123151, arise as a result of the consistent ic/instrument pairing within the 0 123151 VL. Three other features of the passage bear mentioning. First, the 0 123151 VL involve more shared pcs as the passage progresses; that is, at S$_0$ 0372 the 4-13s share two pcs, at W$_0$ 0295 three, and at S$_0$ 0372 all four. This is reflected by gradual decreases in dynamic level, rhythmic activity, and register. Second,
all 4-13s but the fourth are in prime orientation; these articulate $S_9-S_9-S_8-S_0$, which is a large-scale projection of 4-13. Finally, the staggered pc entrances within each 4-13 are calculated to reinforce pitch/rhythmic relationships; that is, each ic 2 in violin II articulates an attack-point duration of 2 (taking the eighth note as the unit), each ic 3 in the viola duration 3, and each ic 5 in the cello duration 4. The simultaneous arrival of the pcs in the final chord strengthens the sense of cadence.

[52] We conclude the discussion of uic type $[x|y|1|z|1]q$ by pointing out uic $1^21415^1$, whose sixty-four VL are the most of any uic we have seen. These sixty-four organize into sixteen Z-pairs, with a further quartet of Z-quadruples in both MORRIS+$^2$ and MORRIS+$^4$. Example 17e realizes the twenty MORRIS+$^2$ VL within this extraordinary uic.

**Part V. Concatenating MORRIS+$w$ VL and Controlling Individual Voices**

[53] Part V focuses on several ways to concatenate MORRIS+$w$ VL while controlling the content of one or more individual voices. First, it concatenates repeated instances of a single VL and alternates Z-related VL. Second, a set of VL within a single uic are put together so that they saturate each voice with a different ic and then so that each voice alternates between two ics. Two further examples saturate a given voice with a particular pc interval (not just interval class) and another shows how to maintain a consistent relationship between two voices.

[54] In order to study the content of individual voices it is crucial to consider the following general feature of VL concatenation: given a series of three chords that articulate VL$_a$ then VL$_b$, ci $x_4$ at matrix position $(r_a,c_a)$ within the matrix of VL$_a$, and ci $x_5$ at matrix position $(r_b,c_b)$ within the matrix of VL$_b$, $ci \times_y will follow ci \times_z in the same voice if and only if c_a = r_b$.

[55] For example, if VL$_a$ and VL$_b$ are both equal to MORRIS+$^2$ S$_2$ 8273, ci 8 at matrix position (1,3) within VL$_a$ will be followed in the same voice by ci 7 at matrix position (3,4) because the column of (1,3) and the row of (3,4) have the same value, 3. Similarly, ci 7 at matrix position (3,4) within VL$_a$ will be followed in the same voice by ci 8 at matrix position (1,3) within VL$_b$, ci 3 at matrix position (4,1) within VL$_a$, and ci 2 at matrix position (2,2) within VL$_b$, and ci 2 at matrix position (2,2) within VL$_a$ will be followed in the same voice by another statement of ci 2 within VL$_b$. Not only does this information give a complete description of the voices for the three chords that articulate VL$_a$–VL$_b$, but it also dictates that additional consecutive applications of MORRIS+$^2$ S$_2$ 8273 will result in three voices saturated with the recurring ci pattern 8–7–3–… and the other voice with repetitions of ci 2. Three consecutive statements of S$_2$ 8273 yields an overall S$_0$ 6666 (because $8 + 7 + 3 = 2 + 2 + 2 = 6$) and six consecutive statements creates an overall S$_0$ 0000, a return to the original chord with the same pcs in the same voice. The four-voice realization in Example 18a illustrates this. The bass, soprano, and tenor voices are saturated with ci-succession 8–7–3. The bass voice is composed of two statements of it (C – A – E – G and G – E – B – C#) and the soprano and tenor voices are transposed rotations of the bass voice. Differentiating itself from the other voices, the ci-2 saturated alto voice articulates a complete whole-tone scale.

[56] Example 18b provides a slightly different situation: twelve consecutive statements of MORRIS+$^2$ S$_2$ 5A89 saturate all four voices with a repeating ci pattern, 5–A–9–8, which creates S$_8$ 8888 every fourth chord and an overall S$_0$ 0000. Example 18c alternates the Z-related VL that appear separately in Examples 18a and 18b. The result is a pair of voices saturated with ci-succession 3–5–2–A (soprano imitated two beats later at T$_4$ by alto), another voice that alternates ci 7 and 9 (bass), and another that repeats ci 8 (tenor). Overall, these sequential patterns create voices that articulate symmetrical pc collections: the augmented triad (3–12[048]), the whole-tone, hexatonic, and enneatonic collections (6–35 [02468A], 6–20[014589], 9–12[01245689A]), and 6–30[013679], which is invariant under $T_6$.

[57] There are a few notable differences when wechsel transformations are involved. First, by definition, the juxtaposition of a VL with its inverse creates an overall S$_0$ 0000, a simple neighbor-like return to the original chord as illustrated by MORRIS+$^1$ W$_B$ 28B9 and B289 in Examples 18d and 18e. Second, consecutive statements of a single W$_n$ create an oscillation between two chords, as with Example 18f's series of W$_B$ 28B9, which alternates D$^6$7 and A$^7$. Highlighting chordal identity, the musical realization places all four voices in the same register so that each D$^6$7 features the same set of pitches as does each A$^7$. Third, any series of wechsel transformations causes an alternation between prime and inverted orientations of the chords, which means that if a voice is saturated with a single ci there is an alternation of inversionally-related pc intervals and therefore an oscillation between two pcs. For example, the series of W$_B$ 28B9 in Example 18f creates
a soprano line saturated with ci 9, which causes an alternation between pc intervals 9 and 3 articulated by C–A–C–A– . . . .

Indeed, any repeating pattern involving an odd number of ci creates abutting inversionally-related segments. For instance, Example 18f’s bass voice features ci succession 2–8–1 twice, once articulated by D–E–A♭–G (pc intervals 2–4–B) and once by G–F–C♯–D (pc intervals A–8–1). Fourth, rotationally-related voices in these wechsel series are related to one another by rotation and T₀, as with the tenor and alto voices which imitate the bass at T₀, two and four chords later, respectively. Finally, Z-triples and quadruples can be used to generate interesting examples such as 18f (and avoid brief neighbor-like figures such as 18d and 18e) as long as inverse VL are not placed side-by-side. For instance, one could arrange the VL in the Z-triple MORRIS+₁ W₁ 11B5/511B/711B into a recurring pattern in which statements of 711B separate the inverses 11B5 and 511B from one another: 11B5–711B–511B–711B.

[58] The general feature of VL connection can also be used to create voices consisting of a single ic. For instance, given the twenty MORRIS+₂ VL within uic 1₁₂₁₄₁₅₁, it is possible to concatenate VL so that each voice is limited to a single ic. Example 19a graphs all such connections. The graph is in two disjoint but congruent halves, dividing the twenty VL into two groups of ten. Single-direction arrows identify situations where one VL may follow another (e.g. W₁ 15A8 may follow W₁ 78AB) and double-direction arrows identify (inverse-related) VL that may precede or follow one another (e.g. W₁ 15A8 and W₁ 8A15). Twelve VL have precisely two VL that could precede it and two that could follow it, four others have two VL that could precede it but only one that could follow it, and the remaining four have only one VL that could precede it but two that could follow. Example 19b provides one path through the left half of the graph, along with a chart that shows that, for each ic, the matrix column value for one VL is the same as the row value for the next. (31) Example 19c provides a four-voice model that articulates the path given in Example 19b. Example 19d provides a short musical excerpt based on the model. The ic₁ voice appears in the flute, the ic₅ voice in the clarinet, and the ic₂ and ic₄ voices in the vibraphone dyads.

[59] Using the same set of VL, it is possible to create alternation between two ics within each voice. The graph in Example 20a illustrates how to concatenate VL so that two voices alternate ics 1 and 2 and the other two alternate ics 4 and 5. The graph is radically different in shape from the previous one: four VL are dead ends and four others are unapproachable beginnings (these eight appear in the center of the graph), and the remaining twelve each have two possible antecedents and two possible consequents that create a complex web with no double-direction arrows. Example 20b provides a four-voice realization for one nine-VL path through the graph and 20c lists ic and matrix position connections for each voice.

[60] We now consider how to control pc interval (not just interval class) within an individual voice. Since having twenty VL within the same uic and MORRIS+₂ is a rare luxury, we study a situation with fewer possible VL, the set of eight MORRIS+₁ schritt transformations that articulate uic 0₁₁₂₅₁ (S₁ 7BA0, S₁ 025₁, S₁ 20B7, S₁ 520₁, S₁ A₁5₀, S₁ A₅7₀, S₁ B₂07, and S₁ A₅₁₀). With these eight VL, even the seemingly modest goal of saturating a single voice with pc interval 5 reaches a dead end after, at most, three VL. One reasonable solution is to permit the featured voice to articulate pc interval 0, but only whenever continuation by pc interval 5 is impossible. The graph in Example 21a illustrates the possibilities for VL concatenation assuming the use of chords in prime orientation. Since the featured voice may contain side-by-side ic 5 but not side-by-side ic 0, it is necessary to track whether the featured voice is articulating ic 5 (e.g. S₁ A₁5₀) or ic 0 (e.g. S₁ A₅₁₀). (32) Examples 21b–d provide musical realizations for three paths within the graph, focusing on S₁ A₁5₀ and S₁ A₅₁₀ and avoiding dead-end nodes such as S₂ 502₁. These series articulate overall S₁ AAAAA, S₈ 8888, and S₈ 6666, respectively, which means that sequential repetition of each string of VL would eventually lead to an overall S₀ 0000. Example 21c shows the current position connections for the featured voice of the path in 21b.

[61] Of course if VL and featured pc interval are carefully chosen, it is possible to saturate a voice with one pc interval with only a few VL and without resorting to interspersed pc interval 0. For example, MORRIS+₁’s W₈ B₁B₁, W₇ A₁B₁, and W₈ B₂A₂ can be combined to create an infinitely long series of pc interval B in one voice—no matter the initial chord’s orientation or featured-voice sm. If the initial chord is in prime orientation ci B will create pc interval B; if sm 1 is in the featured voice then W₈ B₁B₁ is needed, if sm 2 then W₈ 1B₂A, if sm 3 W₈ B₁B₁, and if sm 4 W₇ A₁B₁. If the initial chord is in inverted orientation ci 1 will create pc interval B; if sm 1 is in the featured voice then W₈ B₁B₁ is needed, if sm 2 then W₈ B₁B₁, if sm 3 W₇ A₁B₁, and if sm 4 W₈ B₁B₁. Example 22a presents a graph showing how the string of pc B
There are also many ways to branch out from the VI by a particular ic or ci; for instance, there are eight forty-eight two ic 0, and do not include any other ics. Second, limit VL by transformation; for instance, explore concatenations of the forty-eight MORRIS+5 VI that articulate either W1 or W5. Third, explore situations where one or more set members move by a particular ic or ci; for instance, there are eight MORRIS+2 VI in which sm 1 moves by ci 1 and sm 4 by ci B.

A variety of future directions are suggested. There are other ways to define VOI, voice-leading transformations (VL), and related concepts, and use them to identify and organize the one-to-one and onto voice-leading possibilities for each of four tetrachordal set types. In this context, it presents the MORRIS+5 VL possibilities suggested by Soderberg 1998, demonstrates a few applications, expands the universe to create MORRIS+5a, and shows various ways to concatenate VL while controlling the motion of one or more individual voices. It also categorizes VL in several ways and suggests a variety of compositional applications, in which four-, six-, and twelve-voice pitch-class models yield various monophonic, homophonic, and contrapuntal musical textures, within octatonic, twelve-tone, and other pitch contexts.

This paper defines contextual intervals (ci), ordered/unordered lists of contextual intervals and interval classes (oci, uci, uic), voice-leading transformations (VL), and related concepts, and uses them to identify and organize the one-to-one and onto voice-leading possibilities for each of four tetrachordal set types. In this context, it presents the MORRIS+5 VL possibilities suggested by Soderberg 1998, demonstrates a few applications, expands the universe to create MORRIS+5a, and shows various ways to concatenate VL while controlling the motion of one or more individual voices. It also categorizes VL in several ways and suggests a variety of compositional applications, in which four-, six-, and twelve-voice pitch-class models yield various monophonic, homophonic, and contrapuntal musical textures, within octatonic, twelve-tone, and other pitch contexts.

A variety of future directions are suggested. There are other ways to define voice-leading spaces within the MORRIS+5 universe, including the following: First, gather together VI, with similar (or identical) uic; for example, VI that come from uic 1 3 4 1, 4 3 1, 1 2 4 2, 0 2 1 4 1, 1 2 0 1 4 1, and 4 2 0 1 1 1 include at least one ic 1 and at least one ic 4, may also include one or two ic 0, and do not include any other ics. Second, limit VL by transformation; for instance, explore concatenations of the forty-eight MORRIS+5 VI that articulate either W1 or W5. Third, explore situations where one or more set members move by a particular ic or ci; for instance, there are eight MORRIS+2 VI in which sm 1 moves by ci 1 and sm 4 by ci B. [33]

There are also many ways to branch out from the MORRIS+5 universe. For example, I plan to study the
transformations and VL that arise from moving directly from one MORRIS tetrachord to another; for example, a given member of 4-18 can be followed by any one of twenty-four members of 4-27 and there are twenty-four ways to voice lead each of these, for a total of 576 VL. Such study suggests relating the MORRIS tetrachords and resulting chord progressions to (extended) tonal practice. Also, the VL machinery may be applied to other individual set types. Non-symmetrical tetrachords have the same number of VL (576) but symmetrical ones have fewer. Finally, it is possible to connect the MORRIS tetrachords to others, address sets of other cardinalities, and consider VL that are not one-to-one and onto. All of this, along with the potential analytic and compositional applications of these theoretical possibilities, makes it clear that (tetrachordal) voice leading includes an infinitely interesting set of options that we have only begun to explore.

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Works Cited


Footnotes


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2. Since each MORRIS$_+^\tau$ space includes 576 voice-leading transformations, the number of ways to choose one or more of these possibilities is vast: there are 576 ($= 576!/575!!$) ways to choose one, 165,600 ($= 576!/574!!$) ways to choose two, 31,684,800 ($= 576!/573!!$) ways to choose three, and so on. Coupled with this huge number, the potential rules for concatenation involve an infinite number of ways of requiring, preferring, allowing, or forbidding.

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3. The pair of 4-18 tetrachords provides double-neighbor-like ornamentation of a French sixth chord, which is not part of the MORRIS$_+^\tau$ universe.

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4. Pairs of 4-27 tetrachords connected by ic 1 in each voice are addressed by Cohn 1998, 295.


10. The concept of sm is crucial to a detailed discussion of voice leading and a conventional ordering is a clear way to identify sm. This particular choice of sm and conventional ordering highlights consistencies from one tetrachordal space to another because the diminished triad always involves sm 1–3 and the “warped pc” sm 4. It would have been possible to take sm from the familiar set-class prime forms—and I tried this in my first drafts of this work—but such a system conceals inter-space consistency because of the varying placement of the diminished triad. That is, the diminished triad appears as the second, third and fourth digits in [0258] and [0147] but as the first, third and fourth in [0136] and [0236], and it does not help that the diminished triad variously appears as “036,” “147” and “258.”

11. The W subscripts in Hook’s 2002 study of set class 4-27[0258] reflect the motion of the roots of the major-minor and half-diminished seventh chords, which has the advantage of connecting the W_n to conventional tonal analysis. Hook’s W_n corresponds to my MORRIS+1 W_{[4-n]}. As examples, C7–C#7, C7–C#7, and C7–E#4 articulate Hook’s W_0, W_1, and W_4, respectively, and my MORRIS+1 W_4, W_3, and W_0, respectively.

12. Oci alone is insufficient to define individual voice leading situations. First, some oci appear in multiple MORRIS_w spaces and we need a way to distinguish these situations from one another. For example oci 9003 is part of four different VL: MORRIS1 S_3 9003, MORRIS2 S_3 9003, MORRIS4 S_3 9003 and MORRIS5 S_3 9003. Second, some oci satisfy the harmonic requirements of only one of the four tetrachord types and there is usually no way quick way to determine this (other than trial and error). For example, when applied to a member of 4-13[0136], oci 8259 produces another member of the same set class, but applied to any other MORRIS+_w set type oci 8259 produces a second chord that is of a different set type and therefore not within the scope of this paper.

13. Although the notational conventions of my VL and Lewin’s 1998 “a voice leading from X into Y” (e.g. {A → G♯, D → E, F → E♯}) are somewhat different, the concepts share much in common. Both identify—either explicitly or implicitly—the
chord/set members of X and Y involved in each voice and the melodic intervals they form. The primary difference is that all of my VL are one-to-one and onto whereas some of Lewin’s voice leadings are not. For an approach that defines voice leading by explicitly listing, within each voice, the set member of X, the set member of Y, and the melodic interval, consult Cope 2002, 125.

14. For X = {5, 2, B, 4}, a set in inverted orientation, sm 1 is pc 5, which moves by ci 8 to pc 9, sm 2 is pc 2, which moves by ci 2 to pc 0, sm 3 is pc B, which moves by ci 5 to pc 6, and sm 4 is pc 4, which moves by ci 9 to pc 7.

15. Since the T-matrix features contextual intervals (not pc intervals) any pair of sets that articulate the pc set transformation—no matter the orientation—generates the same matrix. For example, X = {6, 3, 0, 5} and Y = {7, A, 1, 8}, which also articulate MORRIS+W, would also generate the matrix in Example 3b, keeping in mind that in this case the contextual intervals are the inverses of the pc intervals.

By definition, all VL in this paper involve each pc in each set exactly once, but the matrix gives all voice leading possibilities, including those that double and/or omit pcs from one or both sets. For instance, with looser restrictions one could consider the set of voices {0–5, 3–8, 6–B, 3–A}, which includes each sm of Y precisely once but which doubles sm 2 of X and omits sm 4 of X and which articulates ci 5 @ (1, 3), (2, 2), and (3, 1), and ci 7 @ (2, 4).

16. A more detailed explanation of inverses: For each ci at matrix position (j, k) within a given wechsel VL, the inverse VL includes the same ci at matrix position (k, j), which ensures that, if the VL are juxtaposed, both instances of the ci appear in the same voice. Since wechsel transformations alternate prime and inverted orientations, consecutive instances of the same ci in a given voice result in an overall ci 0. For example, ci 8 appears at (1, 2) in W5 8259 but at (2, 1) in W5 8529, which means that, in the example, the soprano voice moves by ci 8 from sm 1 (pc 0) of the first chord to sm 2 (pc 8) of the next chord and then by ci 8 back to sm 1 (pc 0) of the third chord. For inverse-related schritt VL, the row/column swapping works the same way but, since schritt transformations do not alternate orientations, inverse transformations and ci are involved.

17. For more practice with the notions of S/W, VL, uic and inverse, the reader may return to the musical excerpts in Example 2, which articulate MORRIS2 S6 020A (uic 022, MORRIS+1 W1 111B (uic 1'), MORRIS+2 W1 8AA8 (uic 224), and MORRIS+W W1 73A0 (uic 0123151), respectively. Example 2c's MORRIS+W W1 8AA8 is its own inverse, which means that the chordal alternation in the excerpt articulates side-by-side statements of this VL.

18. Within each MORRIS2, S3 w000(−w) and S0 000(−w) are inverses because applying one then the other results in an overall S0 0000. The remaining VL are involutions because they are their own inverses and so applying the VL twice yields an overall S0 0000.

19. The term “obverse” is borrowed from Morris 1998, 185, where it is used to organize six triadic transformations into three pairs (P/P', L/L', and R/R').

20. To see why contrary motion is forbidden, consider starting with {0, 3, 6, B} and have two voices move in contrary motion. For example, if pc 0 moves to pc 2 and pc 6 moves to pc 4 a dead end is reached because there will be no way to reach pcs 8 and A via ic 2 motion without first returning to a previously left pc (0 or 6), which is forbidden. Another example, again starting with {0, 3, 6, B}: if pc 0 moves to pc 2 and pc B moves to 9, creating {2, 3, 6, 9}, one could then move pc 3 to 5 and 6 to 8, but with the B–9 and 3–5 voices moving in opposite directions, pc 1 becomes unreachable.
21. The complete list of series: \(W_3 \rightarrow W_5 \rightarrow W_4, \ W_3 \rightarrow W_5 \rightarrow W_9 \rightarrow W_8, \ W_3 \rightarrow W_5 \rightarrow W_6 \rightarrow W_8, \ W_3 \rightarrow W_5 \rightarrow W_4 \rightarrow W_7, \ W_3 \rightarrow W_5 \rightarrow W_8 \rightarrow W_9, \ W_3 \rightarrow W_5 \rightarrow W_9 \rightarrow W_8, \ W_3 \rightarrow W_5 \rightarrow W_4 \rightarrow W_7, \ W_3 \rightarrow W_5 \rightarrow W_6 \rightarrow W_8, \ W_3 \rightarrow W_5 \rightarrow W_9 \rightarrow W_7, \ W_3 \rightarrow W_5 \rightarrow W_6 \rightarrow W_9, \ W_3 \rightarrow W_5 \rightarrow W_9 \rightarrow W_7, \ W_3 \rightarrow W_5 \rightarrow W_6 \rightarrow W_9, \ W_3 \rightarrow W_5 \rightarrow W_9 \rightarrow W_6, \ W_3 \rightarrow W_5 \rightarrow W_9 \rightarrow W_8, \ W_3 \rightarrow W_5 \rightarrow W_9 \rightarrow W_7, \ W_3 \rightarrow W_5 \rightarrow W_9 \rightarrow W_6, \ W_3 \rightarrow W_5 \rightarrow W_9 \rightarrow W_8, \ \text{and all of their retrogrades.}\) Such series can also be constructed in other MORRIS\(_w\) series, such as \(\text{MORRIS}_1\)'s \(D^\#7-D^7-C^\#6-E^6-D^7-C^\#6\) which articulates \(W_4 \rightarrow W_6 \rightarrow W_7 \rightarrow W_6\).

22. The circle-of-fifths transformation, which amounts to pc multiplication by 5, “maps the chromatic scale onto the circle of fifths and vice versa” (Mead 1994, 36).

23. Such VL exhibit “entirely uniform” voice leading, as defined by Straus 2003, 315.

24. Two notes on this terminology. First, these “groups” are not, strictly speaking, mathematical groups. Second, notice that the third schritt group employs \(w\) in its subscripts whereas the third wechsel group employs \(\omega\). This apparent inconsistency is vital because it facilitates the discussion of similarities between transformations in different MORRIS\(_{w}\). For instance, \(W_{2w}\) refers simultaneously to four transformations—MORRIS\(_1\) \(W_2\), MORRIS\(_2\) \(W_4\), and MORRIS\(_3\) \(W_8\), MORRIS\(_4\) \(W_9\)—which share important voice leading features with one another. For the same reason, subscripts including \(-w\) are necessary in the third schritt group.

25. The other matrix position sets are \{(1, 4), (2, 3), (3, 2), (4, 1)\}, \{(1, 3), (2, 4), (3, 1), (4, 2)\}, and \{(1, 2), (2, 1), (3, 4), (4, 3)\}.

26. Category numbering is arbitrary; that is, category 1 in Example 13c does not correspond to category 1 in other examples.

27. This is true because any pair of dyads has at least two sm interpretations and each sm interpretation has at least one VL that articulates it. For example, given the voice pair \(\{D^\flat-B, C-D\}\), \(C\) and \(D^\flat\) can be interpreted as sm 1 and sm 4, respectively, of a 4-13 in prime orientation, or as sm 4 and sm 1, respectively, of an inverted 4-13. Given the former interpretation, sm 1 (\(C\)) must move by \(ci\,2\) (to pc \(D\)) and sm 4 (\(D^\flat\)) must move by \(ci\,A\) (to pc \(B\)) and so a VL with oci \(2_A\) \(A\) is needed, either \(W_8\,2A2A\) or \(W_B\,2AA\). Given the latter sm interpretation, sm 4 (\(C\)) must move by \(ci\,A\) (to pc \(D\)) and sm 1 (\(D^\flat\)) must move by \(ci\,2\) (to pc \(B\)) and so, once again a VL with oci \(2_A\) \(A\) is needed, either \(W_8\,2A2A\) or \(W_B\,2AA\).
32. With all of the chords in prime orientation and the featured voice stating pc interval 5, S₁ 7Bₐ₀, S₂ 20B₁, Sₐ B₇A₀, and S₅ B₂₀₀ are of no use. Moreover, S₅ B₂₀₀ is useless because no VI have 5 in matrix column 3, which leaves them unable to connect to S₅ B₂₀₀, whose 0 is in matrix row 3.

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33. These are S₁ 140B, S₇ 131B, Sₐ 16AB, Sₐ 113B, W₇ 1B₁B, W₇ 148B, Wₐ 11BB, and Wₐ 12AB.

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34. For example, any 4-12 in prime orientation can be realized as a dominant seventh chord with missing fifth and added minor ninth (C–E–B♭–D♭ ordered low to high) or as a minor triad with missing fifth and added major ninth and augmented eleventh (B♭–D♭–C–E). Another example, the set of voices {C–C, F–E, A♭–G, B–B♭} articulates 4-18 then 4-27, or, in tonal terms, V₇ of F minor with appoggiaturas that articulate an incomplete applied vii°₇, which is especially clearly related to conventional tonal practice if C–C appears in the bass voice.

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