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# Category Structures and Fuzzy Sets

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[1] Prior to his examination of the categorical structure of the first main theme in the first movement of Bruckner's Sixth Symphony, Lawrence Zbikowski comments that "The complexity of the phenomenon of typicality may be one reason Zadeh's theory of fuzzy sets (1) has met with only limited success in characterizing the structure of Type 1 categories." (2) The following look at Prof. Zbikowski's analysis highlights (in a non-technical manner) those aspects of it which lend themselves more readily to "fuzzy" rather than "crisp" models, the complexity of their interaction notwithstanding. In particular, I focus on the problems of describing formally those intuitions which enable us to associate different musical phrases in order to group them as members of the same category.

[2] For the benefit of those persons unacquainted with fuzzy sets, algorithms, arithmetic, measures, etc., I'd like to emphasize that I use the word "fuzzy" as a connotationally neutral term which refers simply to matters of "to what degree?" (graded membership) rather than "yes, or no?" (exclusive membership). (3) Whenever I refer below to aspects of "fuzziness" in Zbikowski's analysis, I'm *not* using the word in its colloquial, pejorative sense, to criticize the vague handling of ideas which one ought to render precisely. Rather, I elaborate on various points where, implicitly or explicitly, he touches on the intrinsically imprecise character of certain musical ideas. Thus, I'm not taking issue with his analysis; on the contrary, I merely offer observations geared toward an appropriate formalization of some of the comparatively informal insights which he offers (all of which I believe to be valid).

[3] At first glance, Zbikowski seems to be dealing with a classic fuzzy set: for elements in a universe of "crisply" defined segments of the Bruckner, he tests for membership in the set of *typical* instances of the first theme. The set is fuzzy because it admits degrees of membership: we don't ask simply

> "*Is* this a typical instance of the first theme (or not)?",

but rather,

> "*To what degree* is this instance of the first theme typical?"

Accordingly, Zbikowski ranks these instances: taking measures 2–6 as typical, measures 8–12 are “somewhat less typical”(4), measures 26–28 and 32–34 are “even less typical,”(5) and so on. It’s a simple step to formalize these rankings by translating them into membership values between 0 and 1, which reflect not only their relative order but the subjective distances among them as well.(6)

[4] However, as Zbikowski points out, the process by which we assign these degrees of membership is anything but linear. Discussing the fragmentary gestures in measures 35–36, he notes that our judgments of typicality are not simple tallies of the numbers of structural propositions met (and the degree to which a musical segment fulfills them).(7) Instead, some propositions may carry more weight than others, such as his P4, by which texture can signal thematic potential, and these relative weights may well vary according to context.(8) Such factors suggest fuzzy algorithms(9) to model our intuitive calculations of typicality, for example:

> “If the texture is *highly* complex, then assign *much less* weight to correspondences of rhythm and contour.”(10)

[5] Additionally, examination of the structural propositions reveals varying degrees of specificity. Consider Zbikowski’s P3, which describes the contour of the theme’s first statement. As Zbikowski compares it to subsequent statements, many of the contours stated as specific (chromatic) intervals seem better served by generic (diatonic) intervals, as in his Figure 1:

specific intervals:
mm. 2–6: 0 -7 -2 +2 +1 -1 -2 +2 +8 -1
mm. 8–12: +1 -7 +1 -1 -2 +2 +1 +1 +7 +1

generic intervals:
mm. 2–6: 0 -4 -1 +1 +1 -1 -1 +1 +5 -1
mm. 8–12: +1 -4 +1 -1 -1 +1 +1 “0” +4 +1

[6] But though these generic intervals clarify the diatonically precise inverse relation of the underlined segments, they fail to convey other essential information: the directed specific interval of -7 between the first two structural tones (Zbikowski’s P2), the semitone at the very end, and the presence of a semitone in the turn figure, the absence of which, as Zbikowski mentions, diminishes the typicality of the statements in measures 159–66.(11) Furthermore, one is left with the task of accounting for the discrepancies between the first, eighth, and ninth intervals of the two series: are we to admit any “off-by-one” surrogate as an acceptably similar variant? If so, are we to admit any number of such surrogates, or is there a point at which the accumulated alterations would disrupt our sense of parallelism? In his pursuit of larger issues, Zbikowski declines to treat these details, yet they are decidedly nontrivial in the creation of an explicit formalization.

[7] So, in addition to (*not* “instead of”) Zbikowski’s P3, we might propose a slightly more general (and thus more robust) version of the theme’s contour:

P3'a) the theme contains four structural notes,

P3'b) the first (preceded by a sixteenth-note pickup) descends to the second by a perfect fifth,

P3'c) a stepwise diatonic turn figure (incorporating a semitone) embellishes the second structural note, proceeding to the third,

P3'd) the theme then ascends by leap to the penultimate note, which neighbors the final one (i.e. the fourth structural note) by a semitone.

[8] Like Zbikowski’s original P3, this new description (P3') of the theme’s initial contour doesn't rely on a specification of scale step. However, P3' incorporates both more and less detail: more, in that it invokes a local diatonic frame of reference and (implicitly) considerations of harmony (structural and embellishing tones); less, in that it doesn't specify the interval of the pickup in P3'b, the direction of the turn in P3'c, nor the size of the leap in P3'd. I offer P3' as a version of how one might

generalize from Zbikowski's P3 after hearing measures 8–12: measures 2–6 are still *more* typical, but P3' subsumes the points by which the second statement deviates from the first. The modified description thus captures somewhat more explicitly the ways in which the statement of measures 8–12 is more typical than some of the subsequent variants, but without taking on the full complications of a "binary basis for typicality."(12) Furthermore, it does so in a way that distinguishes among the various stepwise motions: some are intrinsic to the theme's schematic contour, while others appear as the results of local harmonic changes.

[9] The utility of such vague terms as "leap" becomes apparent when we consider the thematic variants beginning in measures 159–62. Again, a comparison based on directed specific intervals (Zbikowski's Fig. 2) seems to indicate as many differences as similarities:

specific intervals:
mm. 2–6: 0 -7 -2 +2 +1 -1 -2 +2 +8 -1
mm. 159–62: +12 +2 -2 -3 +3 +2 -2 -9 +1

Although Zbikowski describes the relation between the contours of measures 2–6 and measures 159–62 as an "exact mirror"(13), we see from the specific intervals that the only "exact" aspect of mirroring is with regard to interval *direction*, not size. Again, conversion to diatonic intervals doesn't resolve all discrepancies:

generic intervals:
mm. 2–6: 0 -4 -1 +1 +1 -1 -1 +1 +5 -1
mm. 159–62: +7 +1 -1 -2 +2 +1 -1 -5 "0"

How, then, can we model the intuition of an "exact mirror"? One answer lies on a still greater level of generalization, in a distinction between large (L) and small (s) intervals--or, colloquially, between (L)eap and (s)tep-or-(s)kip:(14)

fuzzy-generic intervals:
mm. 2–6: 0 -L -s1 +s1 +s2 -s2 -s1 +s1 +L -s
mm. 159–62: +L +s1 -s1 -s2 +s2 +s1 -s1 -L +s

The designations "s1" and "s2" (ideally with subscripted numerals) provide a simple but flexible means to preserve the turn's diatonic character: that is, although the specific manifestations of s1 and s2 may vary among theme statements, they must adhere to fixed intervals within a single statement.(15)

[10] Not surprisingly, the variants in measures 163–82 require further adjustment to our description:

specific intervals:
mm. 159–62: +12 +2 -2 -3 +3 +2 -2 -9 +1
mm. 163–66: 0 +12 +2 -2 -3 +3 +3 -1 -11 -1
mm. 167–70: 0 +12 +1 -1 -4 +4 +1 -1 -7 -1
mm. 171–74: 0 +12 +1 -1 -4 +4 +1 -1 -7 +5
mm. 175–78: 0 +12 +2 -2 -3 +3 +2 -2 -9 +1
mm. 179–82: 0 +12 +2 -2 -3 +3 +2 -1 -9 +1

generic intervals:
mm. 159–62: +7 +1 -1 -2 +2 +1 -1 -5 "0"
mm. 163–66: 0 +7 +1 -1 -2 +2 +2 -1 -6 -1
mm. 167–70: 0 +7 +1 -1 -2 +2 +1 -1 -4 -1
mm. 171–74: 0 +7 +1 -1 -2 +2 +1 -1 -4 +3

mm. 175–78: 0 +7 +1 -1 -2 +2 +1 -1  -5 “0”

mm. 179–82: 0 +7 +1 -1 -2 +2 +1 “0” -6 +1

fuzzy-generic intervals:

mm. 159–62:  +L +s1 -s1 -s2 +s2 +s1 -s1 -L +s

mm. 163–66: 0 +L +s1 -s1 -s2 +s2 +s2 -s  -L -s

mm. 167–70: 0 +L +s1 -s1 -s2 +s2 +s1 -s1 -L -s

mm. 171–74: 0 +L +s1 -s1 -s2 +s2 +s1 -s1 -L +L

mm. 175–78: 0 +L +s1 -s1 -s2 +s2 +s1 -s1 -L +s

mm. 179–82: 0 +L +s1 -s1 -s2 +s2 +s1 -s  -L +s

Although relaxing the “s1” restriction on the antepenultimate interval resolves some discrepancies, the anomalous endings of measures 163–66 and measures 171–74 suggest the need for a fuzzy corollary: “Correspondences toward the beginnings of statements are more important than those toward their ends.”(16) Indeed, applying this fuzzy corollary to the generic-interval descriptions enables us to recover these as sufficiently general descriptions of the statements in measures 159–82, and thus to invoke the fuzzy-generic level of description only as the means by which we relate these to the theme’s initial (and subsequent) manifestations.

[11] Returning to the verbal descriptions of the theme’s contour, we see that just as Zbikowski’s P3 provides a basis for generalization to P3' in order to encompass the theme’s first two statements, P3' can in turn serve as the basis for an even more general version which bridges the gap between Zbikowski’s first and second categories (17):

P3"a) the theme contains four structural notes,

P3"b) the first (normally preceded by a sixteenth-note pickup) moves to the second by leap,

P3"c) a diatonic turn figure embellishes the second structural note, proceeding to the third,

P3"d) the theme then moves by leap (opposite the direction of the initial leap) to the penultimate note, which normally neighbors the final one (i.e. the fourth structural note) by a semitone.

P3" thus incorporates a high level of generalization of the theme’s contour across its various incarnations, to provide a description which one can fine-tune to produce either Zbikowski’s P3n (measures 159–182) or my own P3' (measures 2–6, 8–12), which one can further refine to arrive at Zbikowski’s original P3 (measures 2–6). The nesting of generalities within generalities suggests that a successful numerical formalization of Zbikowski’s analysis would not only require fuzzy algorithms, but that some of the quantities involved might themselves be fuzzy: that is, not a unitary value between 0 and 1, but a weighted *range* of values between 0 and 1. And, as Zbikowski emphasizes, the dynamic process by which we interpret Bruckner’s “system of approximate correspondences *and* exact correspondences” (18) dictates that these values (and, indeed, their relative degrees of precision) would vary over time in order to model appropriately our aural cognitions.

[12] Again, none of the above is to dispute Zbikowski’s findings; rather, I wish to focus attention on the many different degrees of precision that come into play in the course of his analysis. In particular, we see that there is no single “proper” or “correct” level of specificity in describing our intuitions about thematic character, especially those by which we associate disparate material. It seems self-evident that, as competent listeners, we perceive music neither as a great blur of shadowy patterns nor as an object of real-time, exhaustively detailed note-for-note (interval-for-interval, duration-for-duration, chord-for-chord . . . ) analysis. Instead, we infer chunks of versatile structure, such that we not only register exact matches but also distinguish among transformations which run the gamut from the slyly (or even cryptically) subtle to the boldly (and even shockingly) dramatic. For these reasons, formalisms based on fuzzy sets and fuzzy algorithms seem ideally suited to model the respects in which, as Zbikowski notes, “categorical processes adequate to music must deal with a large amount of auditory information streaming by in real time . . . [requiring] a model of categorization that is extremely rapid (at least in its gross aspects) and highly flexible.”(19)

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## Footnotes

1. Lotfi A. Zadeh, "Fuzzy sets," *Information and Control* 8 (1965): 338-353.
Return to text

2. Lawrence Zbikowski, "Theories of categorization and theories of music," *Music Theory Online* 1.4 (1995): 11.
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3. Zadeh, "Fuzzy sets," 339.
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4. Zbikowski, "Theories," 17.
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5. Ibid., 18.
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6. Zadeh, "Fuzzy sets," 339.
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7. Zbikowski, "Theories," 18.
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8. Ibid., 25.
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9. Lotfi A. Zadeh, "Fuzzy algorithms," *Information and Control* 12 (1968): 94–102.
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10. Construction of an explicit fuzzy model would then entail mapping the imprecise quantifiers "highly" and "much less" to ranges of appropriate numerical values. (see Zadeh, "Fuzzy algorithms," 96.)
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11. Zbikowski, "Theories," 22. Zbikowski doesn't comment on whether he feels that the semitones in the statements of mm. 167–74 somehow impart greater typicality to those variants in relation to mm. 159–66 and 175–82. My own intuition is that the presence or absence of a semitone is actually subordinate to the distinction between a strictly stepwise turn (mm. 2–6 and

mm. 8–12) and one which embellishes by step-above and third-below (mm. 159–82).
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12. Zbikowski, "Theories," 23.
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13. Zbikowski, "Theories," 20.
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14. Although not needed within the limits of the present narrow example, a full-fledged fuzzy model would treat explicitly the vague transition between "large steps" and "small leaps."
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15. These restrictions thus prevent inappropriately chromatic realizations of

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        <+s s s +s +s s>
such as <+3 2 2 +1 +2 1>
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--a perfectly good turn for Ligeti or Birtwistle, but obviously not characteristic of Bruckner's material at hand.
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16. The corollary is fuzzy in that it does not specify hard- and-fast boundaries for which notes do or do not belong to the "beginning" and "end" of a segment--instead, the saliences of correspondences decrease gradually in sequence.
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17. Zbikowski, "Theories," 21.
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18. Ibid., 25.
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19. Zbikowski, "Theories," 26.
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