a journal of the Society for Music Theory

Volume 2, Number 6, September 1996

Intonational Injustice: A Defense of Just Intonation in the Performance of Renaissance Polyphony

Jonathan Walker

KEYWORDS: Just Intonation, Syntonic Comma,1/4-Comma Mean-Tone Tuning, Ptolemy, Josquin, Bartolomé Ramos, Zarlino, Bent

ABSTRACT: Just intonation is commonly dismissed as an impractical, utopian system which could not have had any role to play in the performance of Renaissance vocal polyphony. In the present essay, I attempt to refute this item of received wisdom by providing a theoretical model of a practicable just intonation system, a model which overcomes many of the traditional objections through its flexibility and pragmatism . I argue that other objections result from the inappropriate imposition of a keyboard-based model of pitch on a vocal repertoire. The second section of the essay is devoted to an historical consideration of the counter-claims of Pythagoreanism as the correct intonational system for this repertoire; I argue why there is good reason to believe that a gulf developed between intonation as described by most theorists up to the late 15th century, and intonation in compositional and performance practice.

Prefatory Note on Notation

In order to minimize the number of GIF files for this article, I shall be using notations that remain within the ASCII character-set; while these are probably familiar to the larger part of the readership, they may nevertheless remain unknown to some. The character "^" as in "2^5," is to be read "to the power of." Since ASCII does not include the root sign, I shall be employing an equivalent notation, thus "5^(1/4)" is to be read "the fourth root of 5," while "2^(7/12)" reads "the twelfth root of two-to-the-power-of-seven." "2^-1," "two to the power of minus 1" is equivalent to "1/2," so "2^-2" is equivalent to "1/(2^2)." Having clarified these details, we can proceed to the main body of the article.

Introduction

"Just intonation can never have had any real practical application in vocal music."

—Roger Wibberley [1]

"Just intonation is a utopia."

—Nicolas Meeus [2]

[1] Just intonation has long been considered a theoretical chimera which could have had no role to play in the performance of Renaissance polyphony. In conscious opposition to this consensus, I wish to argue here that the standard presentation of just intonation by its opponents (and sometimes its misguided supporters) is a straw man, and that a more flexible and nuanced system of just intonation (hereinafter JI) is indeed viable, and suffers from none of the obvious shortcomings present in its straw-man counterpart. While for reasons of space, the main burden of my argument here will be to demonstrate that JI is not an impossible, nor even an impracticable system, I shall nevertheless sketch out some reasons why we should also consider it historically correct. Section I, below, is a largely theoretical exposition of the JI system I wish to defend, while Section II provides evidence to latch theoretical possibility onto historical actuality.

[2] For the purposes of this discussion, I shall first introduce a convenient categorization of the possible approaches to tuning theory that have been evident from Greek antiquity to the present. The first approach I shall call the Pythagorean: this clings dogmatically—even mystically—to the number 3 as the basis for musical order; musical intervals should be constructed, accordingly with ratios involving primes no higher than 3. [3] This includes Pythagoras himself, of course, and the Pythagorean cult that grew up among his disciples, but it was also inherited, via Boethius, by the medieval Church, which invested pagan numerology with a new Trinitarian significance; Pythagoreanism is still manifest, perhaps not so mystical, but still dogmatic, in contemporary musicology, not only among specialists in medieval and Renaissance European music, [4] but also, occasionally, in ethnomusicology. [5]

[3] The second approach I shall call the Aristoxenean, after Aristoxenus the deflator of Pythagoreanism, who argued that musical intervals are very much a subjective and arbitrary matter; he also passes muster as the first to adumbrate tempered systems, although this arose partly through his mathematical incompetence, since the Greeks possessed no mathematics of the irrational numbers (Euclid was quick to refute Aristoxenus). [6] Aristoxeneanism is manifest today among those who believe that tuning systems are necessarily and exclusively cultural constructs, based on the accumulation of subjective preferences; [7] I shall include within its compass those who are wont to use the 12-note equal tempered division of the octave as their arbitrary norm for describing other tuning systems.

[4] The third approach provides a via media between these two extremes: Ptolemy rejected both the dogma of Pythagoras and the pure subjectivism of Aristoxenus, describing the Greek genera in terms of the lowest-numbered ratios he could judge, by ear, to correspond to the intervals used by musicians; [8] for example, he replaced the Pythagorean description of the major third (to use our term) “81/64” with the simpler ratio 5/4, which he judged to be in accordance with practice. His approach was therefore to account for practice without abandoning the precision of ratio terminology, but to use this terminology without the prescriptivism of the Pythagoreans. My own position is thus Ptolemaic. I shall call upon these three categories wherever they may serve to clarify the argument, but I would ask readers to remember that I am using them as nothing more than a convenience—not as part of a grand thesis uniting Ancient Greek and Renaissance music theory beyond what is historically warranted.

I

[I.1] I shall now move on to the expository discussion of JI, as a practicable tuning system. (I would suggest that readers already familiar with the basics of tuning should pass rapidly through this and the next paragraph; I prefer not to omit these elementary principles, lest the discussion become incomprehensible to a few valued readers). In order to avoid ambiguity, I shall introduce new terminology for the two ratio-based tuning systems described in this section. The octave is, of course, found on the monochord at the point which divides it in two, and successive octaves are produced by dividing the string into 4, 8, 16, 32 parts, etc. All octaves are thus found in proportions involving powers of two only—this introduces the first prime. In order to obtain fifths, another prime, 3, must be introduced, hence the perfect fifth, 3/2, the perfect fourth, 4/3, and the major second, 9/8. In each case the numerator and denominator is always either a power of 2 or 3.

[I.2] If we take C as an arbitrary starting point, which we shall call 1/1, then G, D, A, E and B, all within the same octave, will be expressible in ratio terms as 3/2, 9/8, 27/16, 81/64 and 243/128. All the ratios are thus located between the 1/1 starting point and 2/1, the octave; while the numerator ascends in powers of 3, the denominator ascends in powers of 2 such that the denominator is always less than the numerator, but greater than half the numerator. Travelling in the other direction from C, the notes F, B♭ and E♭, again moved up within the same octave, have the ratios 4/3, 16/9 and 32/27—the denominator now contains the power of 3, and the numerator is greater than the denominator, but less than half its value. Such a tuning system, based purely on ratios which include only the first two prime numbers and their powers, is most often called Pythagorean (and this accords with Pythagoras' teaching) but the term is ambiguous, if only because the Aristoxenean tendency prefers not to distinguish its two categories of opponent, labelling them all Pythagorean; in any case such a system was already known to the Chinese two millenia before Pythagoras, so it would seem incorrect to call them Pythagoreans. I propose, therefore, a neutral term which carries no baggage: "3-limit system."(9) This also has the advantage of being self-explanatory and easily remembered, now that its context has been set.

[I.3] What then of the next prime number, 5, and a corresponding 5-limit system? Using the same principles as above, but now with powers of 2 and 5 forming the ratios, we will take C again as 1/1 , and rise by major thirds to E and G♯—these are expressible in ratios as 5/4 and 25/16, with powers of 5 in the numerator. Descending by major thirds, A♭ and F♭ will be the ratios 8/5 and 32/25. You will observe that the so-called "enharmonic equivalents" of equal temperament are no longer equivalent, since the 5-limit E, 5/4, and F♭, 32/25, are patently not one and the same pitch. But of still greater importance for this discussion is the comparison between this E, and the 3-limit E we had mentioned above: these Es are also distinct, since the 3-limit interval between E and C is 81/64, while the 5-limit interval is 5/4. Which E is higher? If we express both in terms of a common denominator, 64, the 5/4 becomes 80/64, which is clearly a narrower interval than its 3-limit counterpart. In order to find the interval between the two Es, we must divide 81/64 by 80/64; the 64 cancels out, leaving us with the ratio 81/80. The interval expressed by this ratio is the syntonic comma, and it is this which shall be the focus of all my arguments.

[I.4] I would invite readers at this point to examine the right-hand section of **Figure 1**, which displays a fragment of the 5-limit system.(10) All ratios are expressed in relation to C, and lie within the span of one octave. The letter-name lattice on the left represents the same set of intervals as the ratio lattice on the right. Perfect fifths (3/2) lie along the verticals; just thirds (5/4) lie along the horizontals. "+" raises a note by one syntonic comma (81/80); "-" lowers a note by one syntonic comma. "0" cancels comma inflections. All ratios which do not lie on the horizontal and vertical axes passing through 1/1 (C) combine the primes 3 and 5; for example, to arrive at B, we add a 5-limit major third to a perfect fifth, i.e. 3/2 x 5/4 = 15/8 (remember for other intervals the power of 2 requires adjusting to keep the interval within the same octave. Note that while the 3-limit system is a proper subset of the 5-limit system, I shall use, for convenience, the name "5-limit interval" for those intervals that are not also members of the 3-limit. Both systems are, for theoretical purposes, infinite (which does not prevent one from being a proper subset of the other). The shaded area is the Ptolemy Sequence (as discussed by Zarlino) on C; this area can equally well be regarded as the C major scale, or the Ionian and Hypoionian modes—the identical intervallic structure does not imply identity with respect to any other properties. All the church modes, including Glarean's, together with the major and minor modes are located within the group of letters that bear no syntonic comma or chroma (i.e. flat/sharp) inflections—the final of each mode then takes the place of C as 1/1.

[I.5] Just as the Pythagorean comma (3^12/2^19 = 531441/524288) arises through the incommensurabilty of the "2-limit" (i.e. octaves) and the 3-limit, so the syntonic comma arises through the incommensurability of the 3-limit and the 5-limit. To translate these into cents, the Pythagorean comma is roughly 23.5, while the syntonic comma is roughly 21.5 (cents equivalents will always be approximate, except for equal temperament, from which the measure is derived—a cent is the irrational proportion 2^[1/1200] to 1). While the syntonic comma is therefore too small to be of melodic significance in a musical tradition such as Renaissance vocal polyphony, it is by no means too small to lack harmonic significance. The syntonic comma arises through the conflict between the accurate tuning of 3/2 fifths and 5/4 major thirds, and similarly between the accurate tuning of any 3-limit and 5-limit intervals. Referring to Figure 1 again, observe the interval between C and C♯, represented by 25/24, the 5-limit chroma (i.e. the inflection notated by means of a sharp or flat). This is, of course, a melodically significant difference. In general, Renaissance polyphony involves ficta sharps only at cadences, while flats may be required at any point, to avoid undesirable harmonic or melodic tritones; melodically significant chromas are thus restricted to the requirements of ficta practices. What of syntonic commas? The case I wish to make here is that they could be used freely, either as upwards or downwards inflections, since being melodically insignificant, they were not bound by melodic conventions. Nevertheless, being harmonically significant, they were required in maintaining, as far as possible, the accurate intonation of 3-limit and 5-limit intervals.

[I.6] Since I am not arguing for a rigid, purely theoretical system, but rather a pragmatic, flexible and ad hoc resolution—in practice— of the conflicts between 3-limit and 5-limit intervals (this is what I am presenting as JI), I shall deal now with the possibilities offered by specific examples. The passage I wish to examine is the sequence from Josquin's *Ave Maria* which has been discussed elsewhere, for different theoretical purposes, by Margaret Bent and Roger Wibberley.(11) I shall use this passage firstly to demonstrate how JI can operate in detail, and secondly to demonstrate that JI is, strictly speaking, neutral between the Bent and Wibberley versions, which follow very different courses from each other in the application of ficta chromas.(12)

[I.7] Observe now **Example 1**, which is Wibberley's version of the Josquin, with my syntonic comma inflections added. Notice that in this version, I have used only one extra pitch-class beyond Wibberley's set, namely the D- (Wibberley's B♭s have consistently become B♭-s); all Wibberley's perfect fifths are preserved, while the 5-limit intervals are now also rendered accurately (I realize this was no concern of Wibberley's, but I shall leave this matter until Section II). One of the most important strategies for accurate, but practicable 5-limit intonation is the inflection, mid-way, of a longer note—this occurs in the tenor part, bar 45, and again in bars 46/47; such a move is unavailable where the melodically significant chromas are concerned, but it is generally indispensible for JI realizations, and unavoidable in the present sequence of 5–6 progressions.

[I.8] Notice also that the pitch at the close of the passage is identical to the pitch at the opening, whereas one of the chief objections to JI is the supposedly unavoidable accumulation of syntonic comma shifts downwards. The notion that the comma shift can only draw the pitch downwards is to treat commas, by faulty reasoning, as if they must behave like chromas. Not all simultaneously sounding intervals are given equal weight: for example, the eighth-note neighbor-note D in the cantus, bar 50, need not flatten a syntonic comma to form a 4/3 with the A already sounding in the altus—it passes too rapidly for this to be a consideration; and if the D is not constrained to tune from the A, it will then form a correct 6/5 minor third (plus an octave) with the B neighbor note in the bassus part (and leaving rapidity aside, this would surely be a more important consideration). Likewise the passing-note D in the cantus, bar 52, need not be deflected by the bassus A already sounding, again because of its rapidity.

[I.9] With these considerations in mind, we can examine the very different prognostication of Nicolas Meeus—namely, that this passage will inevitably shift downwards by five syntonic commas in any JI reading.(13) This would indeed be the result if it were not possible to weigh up the different prima facie demands of 3-limit and 5-limit intervals, and of melodic and harmonic considerations, in a word, to employ the rigid, dogmatic model of JI which I said was merely a straw man. If my license to leave rapid neighbor notes and passing notes uninflected were restored, the pitch of the final in bar 53 would shift three commas downwards; and if my license to move freely upwards as well as downwards by comma inflections was also restored, the pitch of the final would remain unaltered in bar 53. What my JI version requires us to abandon is the notion of a given notated pitch remaining fixed in performance; we have seen, in the worked example, that a notated D should fluctuate in performance, between D and D- (respectively 9/8 and 10/9 from C).

[I.10] We may now turn to the version of the same passage that Margaret Bent has produced. Since Bent is quite prepared to countenance an irreversible shifting of pitch (although she doesn't happen to diagnose it in this passage), I shall produce two JI readings of her version: the first (**Example 2**) leaves the pitch of the final unchanged at the end, but involves the usual local fluctuations required by JI; the second (**Example 3**) allows the pitch level to sink, irreversibly, by one syntonic comma. The difference between the first and second readings pivots entirely upon what happens in bar 48: the first employs F+s in the middle parts to form 3/2s with the B♭s in the bassus and cantus; the second features the reverse operation, employing B♭-s to comply with the Fs. These three JI readings (Exx. 1–3) should thus serve to demonstrate the neutrality of JI, not only with regard to the Wibberley and Bent versions, but also on the issue of the irreversible sinking of pitch, versus a static overall pitch standard (within which local pitch fluctuations take place); the case for JI can be made independently of these disputes. Again, it should now be evident that the JI system I am describing does not entail that a single correct reading exists for any given passage—on the contrary, it is inevitably pluralistic; indeed, it need not entail that any preference be given to one plausible reading over another.

[I.11] In the foregoing paragraphs, I have presented a *theory* of JI in practice; I am certainly not suggesting that singers would have thought of their intonational practices in this manner. It would even be misleading to imagine that singers concentrated on syntonic comma adjustments -under that description; their concern was merely to maintain good intonation by listening carefully to themselves and to each other, and it is my contention here that this included not only octaves, fifths and fourths, but also 5-limit thirds (5/4, 6/5) and sixths (8/5, 5/3). While I mentioned that JI is not dependent upon any of Bent's arguments on ficta practices, it runs a parallel course; just as Bent argued against Lowinsky's "secret chromatic art" with her counter-thesis of diatonic ficta, so I wish to avoid any appearance of endorsing a "secret microtonal art." Bent's case contra Lowinsky was based upon pointing out the anachronistic thinking which brought Renaissance vocal polyphony within the conceptual framework of modern notation; an Obrecht passage which moves sequentially from F down to F♭ certainly appears exotic in a modern transcription. But Bent argued that this "chromaticism" was only a product of the transcription, since singers (in Obrecht's time) would have been following the purely diatonic contrapuntal implications of the piece (C♭ to C-natural is a chromatic semitone, but C♭ to F♭ is not a chromatic interval at all).(14) Similarly, the addition of "+" and "-" signs to represent syntonic comma "inflections" is purely a matter of achieving clarity within the same framework of modern notation; singers would not have imagined themselves to be applying microtonal adjustments—they would merely have been aiming for good intonation of all the usual intervals.

[I.12] Before moving on to the historical discussion of Section II, I must add a further qualification, lest I be misunderstood: this concerns the degree of precision to be expected of singers. The various possibilities for performance that I have sketched out are merely an indication of what singers might have aimed for; I am not claiming that they were able to do this with extraordinary accuracy, but simply with enough accuracy in general to aim for a 5/4 major third, rather than for a vague "major-third region" which could as easily have resulted in a Pythagorean 81/64. The fact that equal temperament, for all its benefits, has blunted our sensitivity to 5-limit intervals provides us with no reason to imagine that singers in the 15th and 16th centuries were in the same position. Equal temperament, being based on a slight tempering of the 3-limit which shrinks twelve fifths into the compass of seven octaves, will of course reflect the 3-limit quite accurately (e.g. the 3/2 with only about 2 cents discrepancy). The 5-limit is ill-served however, with the 5/4 represented by an interval almost 14 cents wider, and the 6/5 by an interval nearly 16 cents narrower. Our own acceptance of vagueness in the 5-limit is a consequence of equal temperaments ubiquity today; this was entirely reversed, as we shall see below, in the longest-lived system of keyboard tuning, which was in use during the period under discussion: there the 3-limit was compromised for the sake of pure 5/4s. Tempo and the ambient acoustic are also of importance in considering precision: a slower procession of notes and a more resonant environment would afford greater opportunities to realize subtleties of intonation (and conversely, render any intonational inaccuracy all the more obvious). Exx. 1–3 should be treated no more (and no less) literally than these qualifications allow.

II

[II.1] My main concern has been to provide a theoretical case for a practicable JI system; nevertheless, without any corroboration from available historical evidence, such a theory could proceed no further than the realm of—perhaps unrealized—possibilities. I shall now sketch out various reasons which would constrain us to believe that such a JI system was not merely possible, but overwhelmingly probable; for reasons of space, this part of the discussion will be all too brief (I hope to expand upon it on another occasion).

[II.2] The medieval Church inherited its Pythagoreanism from Boethius, and entrenched the number 3 into its account of musical intervals by evoking Trinitarian symbolism in addition to Pythagoras' own numerological obsessions. Consonance was judged the sole preserve of the 2/1, 3/2 and 4/3 (and their counterparts with an added octave-gap); the 81/64 Pythagorean third was understandably excluded as a dissonance.(15) 1, 2, 3 and 4—the elements earth, water, air and fire were four—when added together produced the mystic Pythagorean Dekad, and the number of the Commandments, and so on. Theorists of music (within the quadrivium) were allotted a privileged position over those who merely practiced the art; the theorist was the guardian of a prescriptive, speculative system, and not the empirical investigator of musicians' practices. There was still the occasional flicker of other possibilities: for instance, Hucbald (840–930) described a 5-limit tetrachord—1/1, 10/9, 5/4, 4/3;(16) but the possibility of a scientific enterprise in the spirit of Ptolemy was ignored for the sake of upholding Pythagorean dogma.(17)

[II.3] A distinct problem emerged when chains of thirds were incorporated into two-part organum, and later in the parallel sixths of English faburden. Walter Odington examined these "concords of discord" (i.e. theoretical discord perceived as concord) and found, by means of the monochord, that singers replaced 81/64 and 32/27 with 5/4 and 6/5 respectively; elsewhere, he states that the proportions of the major triad should be 64:81:96:128, thus acknowledging the normativity of Pythagoreanism.[(18)] Franco of Cologne is less timid in admitting thirds: besides "perfect concords" (unison, octave) and "intermediate concord" (fifth, fourth), he provides a third category of "imperfect concord," consisting of major and minor thirds.[(19)] A papal decree issued in 1324 attempted to ban these intervals; it was generally insinuated that they were sensuous, effeminate and worldly. With the passing of another century, the English cultivation of the third—from the "Sumer" canon through to Dunstable and the "contenance angloise"—witnesses eloquently against such dogmatism.

[II.4] By 1482, Bartolomé Ramos was able to sweep away centuries of doctrine with a casual preamble in his *Musica practica* (note the title): "The regular monochord has been subtly divided by Boethius with numbers and measure. But althought this division is useful and pleasant to theorists, to singers it is laborious and difficult to understand. And since we have promised to satisfy both, we shall give a most easy division of the regular monochord."[(20)] Ramos then embarks upon a very lucid description of the 5-limit system, based upon his "easy division" of the monochord, detailing the 5/4 and 6/5 thirds, the 8/5 and 5/3 sixths, and the 16/15 diatonic semitone.[(21)] This work won Ramos much vehement criticism, which continued into the following generation, when his pupil Giovanni Spataro was still being required to defend these monochord divisions. Lodovico Fogliano's *Musica Theoretica* (1529) reinforced and extended Ramos's teaching and by the mid-century, when Zarlino produced his *Istitutioni*, the new ideas were well-established.

[II.5] Leaving aside Zarlino's contribution for a moment, it is worth considering developments in keyboard tuning, which provide irrefragable evidence of the 5-limit intervals' importance in music as practiced. Arnold Schlick's *Spiegel . . . der Organisten* (1511) described the practice of mean-tone tuning which was clearly already in existence; Schlick also provided adequate formulas for obtaining the tuning. Pietro Aron produced a more throrough analysis of the system a decade later, in the *Thoscanella de la Musica*, which sufficed for all practical purposes. Francisco de Salinas finally provided a systematic, mathematical account of the mean-tone system in his *De musica*.

[II.6] In order to ensure that the implications of this development in tuning are clear, I shall look briefly at its main organizing principle. The mean-tone system is based upon the pure 5/4 third; recalling our earlier discussion of the syntonic comma—as the difference between the 3-limit E+, reached by a chain of four fifths, and the 5-limit E—the mean-tone system narrows the chain of fifths so that they equal the span of two octaves and a 5/4 third. Just as the 3-limit is tempered to fit the 2-limit in equal temperament (by removing the twelfth root of the Pythagorean comma from each interval in a chain of twelve fifths), so the 3-limit is tempered to fit the 5-limit in mean tone temperament (by removing the fourth root of the Pythagorean comma from each interval in a chain of four fifths) - hence the full name "quarter-comma mean-tone" (i.e. [81/80]^[1/4]). The fifths, now the narrower 5^(1/4)/1, have thus been sacrificed to maintain the purity of the 5/4 thirds. Remembering that keyboards at this time were principally used to play vocal polyphony (albeit decorated) this should be reason enough for us to concede that the intonation of the third as 5/4 must have been well-established among musicians by the early 16th century—indeed they considered the matter so important that they chose a tuning system which gave this third priority over the fifth. This should also cause us to reflect upon the arbitrariness of our own tuning preference for the fifth to the detriment of the third—such preferences are not immutable and universal, but are based upon historical contingencies, and the power of habit.[(22)]

[II.7] Zarlino is considered the preeminent theorist of JI; *Le Istitutioni harmoniche* (1558) introduces the prime number 5 into the proportions he derives from the senario (1–2–3–4-5–6, in clear opposition to medieval theory): superparticular ratios (i.e. [n+1]/n) from 2/1 to 6/5 were the simplest consonances; "composites" with numerator and denominator up to 6 were next simplest, e.g. 5/3 (composite of superparticulars 4/3 x 5/4); composites with numerator or denominator above 6 were next in rank of consonance.[(23)] Where medieval theorists had learned that 3/2 and 4/3 are the harmonic and arithmetic means between 1/1 and 2/1, Zarlino went on to demonstrate that the 5/4 was the harmonic mean (on a monochord) between 1/1 and 3/2, and the 6/5 the arithmetic (the harmonic mean between x and y is 2xy/(x+y); the arithmetic mean is xy/2). By extension, he assigned "major" tonality to harmonic proportion—a string and its parts (1/1, 1/2, 1/3, 1/4, 1/5, 1/6), while "minor" tonality was derived from arithmetical proportion—equal parts of a string added together (1, 2, 3, 4, 5, 6). Like Fogliano before him, Zarlino championed Ptolemy, and advocated the "Ptolemaic sequence" as the "natural" scale (this consists of the ratios within the shaded area of Fig. 1). Nevertheless, there was a little of the Pythagorean impulse in Zarlino, since he indulged in occasional mystificatory strategies in order to justify his theories: for example, the fact that a cube had only six sides was offered as an argument for stopping at 6 (Mersenne was later to question Zarlino on the issue of this supposedly natural terminus—he suggested that 7 be included in the proportions of musical intervals, a proposal which later interested both Tartini and Euler).

[II.8] Section I argued that just intonation is not merely the useless plaything of theoreticians, but on the contrary is eminently useable when treated as a flexible and pragmatic body of strategies. Section II has supplemented this consideration of JI's possibility with a body of evidence that demonstrates its actuality in Renaissance Europe. While both sections provide no more than a brief sketch of the terrain, I trust they will nevertheless serve to persuade, at least partially, those readers who had formed no fixed opinion on the matter, and to place serious doubts in the minds of those who have hitherto been implacably opposed to JI. I have indicated why the testimony of medieval theorists should not be taken at face-value; the depth of the vested interests at stake became evident as the violent controversy over Ramos' "easy divisions" of the monochord continued for decades—it is easy to see Ramos, without too much exaggeration, as a predecessor of Galileo Galilei, who published his work on heliocentric cosmology over a century later.

[II.9] Perhaps the most satisfying, but at the same time most discomfiting aspect of historical study is the discovery that we have consistently misinterpreted the evidence before us, because we had failed, through a lack of imagination, to enter more fully into another's conceptual framework. In our case, the convenience of transcriptions in modern notation is also a potential handicap, since it renders the task of shedding our assumptions more difficult - assumptions concerning what the score determines, and what differences would have arisen in a musical tradition which lacked scores themselves.(24) The convenience of a tuning system adopted for keyboard instruments in the late 18th century namely equal temperament is a further handicap, which imposes its own conceptual tyranny regarding pitch in general, and tuning systems of a previous age, above all JI, whose every virtue is regarded as a shortcoming within the framework of equal temperament, and vice versa.(25) It is remarkable how blithely we tend to speak of the "circle of fifths," as if the fact that this is merely a product of equal temperament had been cast to the nethermost regions of our memory.(26) We also tend to see equal temperament as the goal of a centuries-long teleological process whereas we ought to be asking why one particular tuning system, and not another, was used for a given repertoire. Need I say that my purpose in this article is not to condemn equal temperament as the intonational decadence that set in after the glorious purity of JI? Of course we should reject such views, but we should equally guard against any notion that the musicians of past centuries were benighted because they lacked equal temperament, or that they were unwittingly striving towards it all the time.(27) As I have already mentioned, equal temperament was the subject of various 16th century experiments, and its mathematics was completely understood b y the early 17th century; composers were not striving towards it—on the contrary, they rejected it.

[II.10] I shall close with some historical considerations that might prove fruitful for future research. Most of the positive historical evidence I have adduced dates only from the late 15th century onwards; given that we need not take the Pythagoreanism of medieval theorists as a faithful description of musical practice (since it was never intended to engage in such inquiry), how far backwards should we project the present arguments for JI? For the tuning of faburden sixths, anything other than the 5-limit intervals is, I believe, highly implausible. But neither would I like to assume that the almost unanimous Pythagoreanism of medieval theorists simply ceased to have any impact on composers. These are qustions for further investigation, which is likely to include not only the examining of musical and other documentary evidence, but also the study through performance of later medieval repertoires in order to discern what intonation would seem most appropriate. By the 16th century, the complete absorption of the 5-limit intervals seems certain, while the 10th century evidence on the intonation of organum thirds is hazy. Assuming that Pythagorean, i.e. purely 3-limit tuning was in fact practiced at some stage (and it is always worth stating our assumptions, even when we don't intend to question them), we may slowly be able to deduce where and when it expanded into the 5-limit system, and where it might have enjoyed a resurgence.

Jonathan Walker
Queen's University Belfast
School of Music

Belfast BT7 1NN
Northern Ireland, U.K.

Footnotes

1. Roger Wibberley, posting to mto-talk, 21 Aug 1996 "Re: Wibberley, MTO 2.5"; the issue of just intonation in Renaissance polyphony was one of the topics of the mto-talk discussion that followed the publication of Wibberley's article "Josquin's *Ave Maria*: Musica Ficta versus mode" in *MTO* 2.5. I should state at the outset that just intonation does not receive any attention in that article.
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2. Nicolas Meeus, posting to mto-talk, 19 Aug 1996, "Re: Wibberley, MTO 2.5."
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3. Andrew Barker, ed. *Greek Musical Writings, Vol. II: Harmonic and Acoustic Theory*, (Cambridge 1989), pp. 28–35. A concise, witty and polemical account of tuning history in the West is to be found in *Genesis of a Music*, Harry Partch (New York, 1949/1974), pp. 361–397.
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4. Wibberley's posting to mto-talk, 21 Aug 1996, in which he defends himself against Meeus's charge that his article assumed JI; in reply, he endorses and articulates a thorough-going Pythagoreanism, and wields this against JI. He states that "[JI's] total incompatibility with medieval and Renaissance theory is manifest in crucial ways." I shall demonstrate later why Renaissance theorists cannot be enlisted en masse to bolster Pythagoreanism.
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5. E.g. Mieczyslaw Kolinski, "The Structure of Music: Diversification versus Restraint," *Ethnomusicology* 22/2 (1978).
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6. Aristoxenus, *Elementa Harmonica*; English translation in Barker, *Greek Musical Writings*, vol. 2, pp. 119–208.
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7. Nicolas Meeus, postings to mto-talk, 19 and 26 Aug 1996. In the latter posting Meeus says, "Roger [Wibberley] writes : 'just intonation can never have had any real *practical* application in vocal music'. Right, but I would be tempted to say exactly the same of the Pythagorean intonation, so many medieval and Renaissance theorists notwithstanding." Meeus also suggests that keyboard tunings are our best source for vocal intonation; part of the purpose of my article is to show that constraints operating upon the closed tuning systems of keyboards (with 12 to 19 notes per octave) had no bearing on unaccompanied vocal music, which allowed infinite flexibility with regard to pitch. Some of Meeus's other comments indicate, however, that we may not entirely disagree.
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8. Ptolemy, *Harmonics*; English translation in Barker, *Greek Musical Writings*, vol. 2, pp. 270–360. I cannot recommend Ptolemy's *Harmonics* too highly, for its content, the quality of the argument, and for its early demonstration of scientific method at work. Here is Barker's endorsement: "The care with which he undertakes this investigation, and his willingness to modify theory in the light of the evidence of practice, give us good reason to trust the accounts he gives. They are precious evidence, unique in their combination of mathematical precision with empirical good faith, about systems of attunement that Greek musicians really used." (p. 272). This would explain in part why he proved so attractive to Renaissance humanist scholars such as Fogliano and Zarlino.
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9. This terminology is to be found in Partch, *Genesis of a Music*; while I owe much to this work, and value Partch's music highly, the present article is not to be regarded as an extension of his polemic, which was motivated by his compositional project.

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10. Fig. 1 is close in form to the 2 and 3-dimensional lattices employed by the U.S. composer-theorist Ben Johnston as a pre-compositional tool; I have freely generalized Johnston's notations for my own theoretical purposes. Cf. Johnston, "Rational Structure in Music," *American Society of University Composers Proceedings* 11/12 (1976–77). The first movement of Johnston's String Quartet No. 2 combines serialism with the 5-limit system; the music is written in such a way that it will ascend quickly by syntonic commas up to the central point, after which it descends in the same manner, back to the original pitch.
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11. Bent, "Diatonic Ficta," *Early Music History* 4 (1984); Wibberley, "Josquin's *Ave Maria*".
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12. As it happens, I am persuaded by Bent's argument in "Diatonic Ficta," and am therefore constrained to reject Wibberley's version and the accompanying arguments; I have further reservations concerning Wibberley's version, above all because of the melodic tritone in the cantus between B♭ and E, bars 48–50. Nevertheless, since this matter is entirely independent of my advocacy of JI, the readings that I present remain scrupulously neutral.
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13. Meeus, mto-talk posting, 19 Aug 1996: ". . . but something odd happens. Consider for instance how the first C in the tenor, m. 44, connects to the second, in the altus, m. 46, by a string of consonances . . . The second C . . . is a syntonic comma lower than the first. . . But the phenomenon repeats: I leave it up to you to check that, on the same premises, the C in the altus in m. 52 is five commas, 110 cents, more than a semitone, lower than the first. . . . Just intonation is a utopia."
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14. Bent, "Diatonic Ficta," especially pp. 34–35.
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15. Cassiodorus, *Institutiones*, translation in *Source Readings in Music History*, ed. Oliver Strunk (London, 1952), p. 89.
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16. FOOTCited in Partch, *Genesis of a Music*, p. 380.NOTE
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17. Cassiodorus nevertheless pays lip-service to Ptolemy's "excellent treatise" in passing; his medieval followers evidently failed to act upon this recommendation. (in Strunk, *Source Readings*, p. 92). A characteristic example of Trinitarian symbolism conjoined with Pythagorean music theory can be found in Jean de Muris, *Ars novae musicae*, in Strunk, *Source Readings*, pp. 173–4. The original text, Muris, *Notitia artis musicae* is to be found in *Corpus scriptorum de Musica*, vol. 17, ed. U. Michels (1972), p. 67.
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18. Cited in Partch, *Genesis of a Music*, p. 372. The original text is to be found in *Corpus scriptorum de musica*, Vol. 14, ed. F. Hammond (1970).
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19. Franco of Cologne, *Ars cantus mensurabilis*, in Strunk, *Source Readings*, p. 153. Franco nevertheless counts sixths among the discords.
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20. Bartolomé Ramos, *Musica practica*, in Strunk, *Source Readings*, p. 201.
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21. Ramos, in Strunk, *Source Readings*, pp. 203, 4. Curiously, the 5-limit chromatic semitone Ramos offers is the 135/128,

which in my Fig. 1 is the interval between C and C♯+. This interval is rather implausible, and I can only imagine that Ramos failed to notice the 25/24 (the interval between C and C♯); it would seem that Ramos may have been Wibberley's source, since he also presents the more cumbersome ratio as the chromatic semitone (mto-talk posting, 21 Aug 1996).
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22. Lest anyone imagine that twelve-note equal temperament would have been adopted instead if only it had been conceived of, it should be pointed out that it is considerably simpler in its mathematics (although the calculations involve more labor). Henricus Grammateus had already drawn up a fairly close approximation in 1518, and Zarlino corrected Vincenzo Galilei's plan for a twelve-stringed equal-tempered lute (Galilei had invoked Aristoxenus as his inspiration in this project). Even though the mathematician and music theorist Mersenne produced a correct and systematic description in 1635, equal temperament was not adopted until 150 years later in Germany and Austria, while Britain and France delayed for over two centuries. In the 17th century, the main alternatives to 1/4-comma meantone were other tempered meantone systems, such as the 1/6-comma, and the 2/7-comma.
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23. The complete text of the *Istitutioni* (in the original Italian) can be obtained from http://www.let.ruu.nl/C+L/wiering/ The introduction, by Claude Palisca, to *On the Modes*, trans. V. Cohen, (New Haven, 1983), provides a useful overview of Zarlino's achievement as a theorist.
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24. Cf. Bent, "Editing Early Music: the dilemma of translation," *Early Music*, August 1994.
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25. The *New Grove* entry on Just Intonation, by Mark Lindley, is conceptually skewed in just this manner: from half-way through the first paragraph, to the end of its two-page spread, keyboard instruments are the sole subject matter (with a passing reference to the guitar, another fixed-pitch instrument). JI's most obvious domain —vocal music—is not touched upon; by contrast, JI is a rather obscure corner in the history of keyboard instruments, with no notable success. As I have argued above, the true keyboard-based progeny of JI was 1/4-comma meantone tuning, which was both very widely used, and enjoyed the longest lifespan of any keyboard tuning to date. It is regrettable that this article should provide the unwary student with a first (and perhaps last) taste of JI.
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26. There are further "circles" after 19, 31, 43 and 53 fifths; Vincentino's archicembalo was variously described by its maker in terms of the 31-fifth "circle," and in terms of ratios derived from Ptolemy, while it can perhaps be understood most convincingly as an extension of the 1/4-comma meantone system.
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27. Lowinsky, "Matthaeus Greiter's *Fortuna*: An Experiment in Chromaticism and in Musical Iconography," *Musical Quarterly* 42 (1956), 43 (1957).
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Prepared by Robert Judd, Manager and Tahirih Motazedian, Editorial Assistant