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Practical Aspects of Marchetto's Tuning

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ABSTRACT: In his historically momentous account of tuning, Marchetto of Padua (fl. 1305–19) proposed dividing the whole tone of received theory (9/8=204 cents) into 5 parts. This report develops determinate arithmetic and geometric realizations of Marchetto's formulation—directly applicable to the medieval monochord, and sonically illustrated by digital files. The resulting intervals' feasibility is compared with current findings in the psychology of interval perception. Conjectures are offered as to how and to whom Marchetto's tuning was taught. A dual formulation of "nuanced," heterogenous tonal systems is advanced to assess structural effects of Marchetto's tuning on pieces and to suggest ways one could learn to perceive and sing his intervals nowadays.

INTRODUCTION

[0.0] Marchetto of Padua (fl. 1305–19) was arguably the most important European music theorist between Guido of Arezzo (fl. ~991/2–1033+) and Gioseffo Zarlino (1517–90). A leading experimental composer of his era, Marchetto taught at Padua's Cathedral, a job that would involve training choirboys, leading the chorus in liturgical chant, performing his own part in written and/or improvised discant (counterpoint), and both performing and rehearsing/directing others in cutting-edge music of his own creation (Gallo 1974: 42–43; Vecchi 1954: 166–68).

[0.1] Under aristocratic patronage, Marchetto published pathbreaking contributions to all the main areas of music theory, then or now: in his Lucidarium (1317–18), musical philosophy, semiotics, numerology, and applied mathematics, as well as more narrowly technical matters of tuning, discant, pitch notation, melodic analysis, and modality; in its sequel, the Pomerium (1318–19), rhythmic theory and notation (Herlinger 1985: 3–21; Gallo 1977/1985: 113–16).

[0.2] These innovations influenced pre-eminent theorists and musicians of Europe for almost 300 years (cf., e.g., Niemöller 1956; Herlinger 1981a, 1981b, 1990). Nonetheless, the novelty for which Marchetto was, and remains, best known—namely, his proposal that whole tones be divided into 5 parts—has been interpreted somewhat indeterminately. E.g., Jan Herlinger's important account concludes (1985: 17):

> Just as Marchetto's enharmonic and diatonic semitones must be approximations, so must be his diesis and chromatic semitone. [The latter] differ from each other in size to a greater degree than [Marchetto's] enharmonic and diatonic semitones, but *just how much* we cannot say [my emphasis].

[0.3] The present report advances determinate interpretations of Marchetto's often difficult account of tuning. Briefly, Marchetto's account favours 2 main readings of his whole-tone division: into 9 or 5 parts. From |Luc. 2.5.15| onward, Marchetto writes uniformly of 5 dieses per whole tone.[(1)] However, to regard the whole tone as divided into 5 parts *tout court* is to discount Marchetto's extended preliminary account of how "the nature of the whole tone, its essence, would consist in the 9-fold number compared to the 8-fold number [i.e., the ratio 9/8]" (quod natura toni et essentia eius consistat in novenario numero ad octonarium comparato: |Luc. 2.4.1–42|) and "in the perfection of the 9-fold number" (in perfectione numeri novenarii: |Luc. 2.5.8|), as well as his theses that:

> a) "the 9-fold number [i.e., as such, in contrast to, e.g., 9 * 2 = 18 or 3 * 3 = 9] can never be divided into equal parts" (novenarius numerus numquam potest dividi in partes equales: |Luc. 2.5.9|);
>
> b) "its parts must be unequal" (partes ipsius debeant esse inequales: |Luc. 2.5.13|);
>
> c) "1 would be its 1st part; from 1 to 3, its 2d [part]; from 3 to 5, its 3d [part]; from 5 to 7, its 4th [part]; from 7 to 9, its 5th [part]; and such a 5th part is the 5th odd number of the 9-fold totality" (unus sit prima pars; de uno ad tres, secunda; de tribus ad quinque, tertia; de quinque ad septem, quarta; de septem ad novem, quinta; et talis quinta pars est quintus numerus impar totius novenarii: |Luc. 2.5.14|)—in other words, that Marchetto construes the 5 unequal parts of the whole tone as comprising 1 + (4 * 2) 9th-parts, a point clarified by his having emphasized that the 9-fold number can never be divided into equal parts, because:
>
> d) "a unit is in it that resists being divided" (est . . . ibi unitas que resistit dividi: |Luc. 2.5.10|), an idea Herlinger astutely connects with Marchetto's later, more complete statement of Remigius s doctrine that:
>
> e) "an even number is mutable and divisible [i.e., into equal segments], whereas an odd number is indivisible, containing a unit *in its middle* that resists division" (numerus par mutabilis et divisibilis est; numerus vero impar indivisibilis est continens unitatem *in medio sui* que divisioni resistit: |Luc. 6.3.14|: [my emphasis]).

[0.4] Although a 5-fold division partitioned into 2 + 1 + 2 would satisfy this last point as well as a 9-fold division partitioned into (2 * 2) + 1 + (2 * 2), it would miss the plausibility that Marchetto regarded as an important aspect of his original formulation of the 9/8 ratio, "not yet discovered demonstrated by writers [on music theory]" (nondum invenitur . . . ab auctoribus demonstratum: |Luc. 2.4.3|), the following idea: each of the 9 parts of the 9th part of any whole tone division is the same size as each of the immediately preceding 8 parts of the 8th part (cf. 81/72 and 72/64, and [0.5], [2.3–5], below). Like other aspects of his tuning, this is something "[he could] display to perception on sonorous bodies, i.e., the monochord etc." (cf.: ostendimus ad sensum in corporibus sonoribus, puta in monocordo et aliis: |Luc. 2.5.8|). It jibes with the earlier medieval privileging of supernumerary ratios, (x + 1) / x, of which the smaller number, x, is a power of 2, i.e., 2^n, as in 2/1 = (1 + (2^0)) / (2^0) (trivially), or more importantly, 3/2 = (1 + (2^1)) / (2^1).

[0.5] Such an approach facilitated geometric construction of ratios by reducing all steps to bisection or addition/transposition of parts resulting from bisection and could be confirmed on the monochord by eye and ear. For Marchetto's novel division, all 9 parts of a whole-tone ratio: 9/8 (= ((1 + 8) / 8) = ((1 + (2^3)) / (2^3))) would be evident in this way, as would all 9 parts within a whole tone: 81/72 (= ((9 * (1 + 8)) / ((8 * (8)) = ((9 * (1 + (2^3)) / ((8 * (2^3))))). That each of the 9 (= 81 - 72) parts in the 9th part of a 9/8 ratio would be of the same length as each of the preceding 8 (= 72 - 64) parts of the preceding, 8th part, which also is a whole tone, since 72/64 = 9/8, would necessarily result from the general inequality: (x^2) - x (x - 1) = x (x - 1) - ((x - 1)^2) + 1 (e.g., (2^2) - 2 (1) = 2 (1) - (1^2) + 1 or 4 - 2 = 2 - 1 + 1 or 2 = 2; (3^2) - 3 (2) = 3 (2) - (2^2) + 1 or 9 - 6 = 6 - 4 + 1) or 3 = 3).

[0.6] The 9 spaces marked off within a whole tone's space would be partitioned into 5 spaces, each of which would be a diesis, the middlemost of which (77/76) would comprise the "unit that cannot be divided:"

```
81   80   79   78   77         76   75   74  73  72
81        79        77         76        74      72
<-diesis-><-diesis-><-diesis-><-diesis-><-diesis->
```

Arithmetically, this would involve dividing the frequency-ratio for the whole tone (e.g., C/D), which for almost 2000 years had been formulated as 9/8:

```
C                                            D
9                                            8
```

into 9 parts (cf. Gurlitt and Eggebrecht 1967: v. 3, 225):

```
C                                            D
9    8    7    6    5    4    3    2    1    0
```

so that the entire whole tone could be understood as 81/72:

```
C                                            D
81   80   79   78   77   76   75   74   73   72
```

Within this newly construed whole tone, Marchetto's most important diesis (e.g., C74/D) would be 74/72 (= 37/36), whereas its complement within the whole tone (e.g., C/C74), would be Marchetto's "chromatic semitone," 81/74: (2)

```
|c|                                |c74|      |d|
81                                 74         72
<--------chromatic semitone--------><--diesis-->
                                   37         36
```

[0.7] Marchetto's other semitones (i.e., parts of a whole tone; incomplete, imperfect tones—not 1/2-tones: cf. |Luc. 2.5.18|) would be, to use further terms he appropriated from Ancient Greek theory (via Boethius),(3) the "enharmonic" semitone, whose 2 dieses would comprise 81/77 (e.g., for A/Bb77) and its whole-tone complement, the 3-diesis, "diatonic semitone" (e.g., Bb77/B), whose ratio would be 77/72 (cf., however, below):

```
|a|                   |bb77|                  |b|
81                     77                     72
<-enharmonic semitone-><---diatonic semitone--->
```

1. OVERVIEW

[1.0] Although Marchetto's numerical formulation is highly interesting in its own right, as is its place in the stylistic, intellectual, cultural, ethnic, and gendered ideological history of Western European music, I focus here on immediate practical aspects of his tuning. Most important, Marchetto's diesis was a very narrow interval, ~48%, i.e., ca. 48 cents, much closer to a tempered quartertone (50%) than to a tempered semitone (100%). With this in mind, the following questions arise:

a) How could Marchetto geometrically draft marks on a monochord to realize his tuning?

b) What physical constraints would a monochord place on conveying Marchetto's tuning:

i) visually?
ii) sonically?

c) How readily could musicians identify Marchettan intervals?

d) How might Marchetto have taught singers to produce reliably and fluently his new intervals?

e) How well would compositions (and discant improvisation) using Marchettan intervals survive the almost inevitable misunderstandings and re-formulations of later centuries?

f) What new conceptual/perceptual understanding would be involved in learning to hear and sing Marchetto's dieses?

[1.1] Briefly, my answers are as follow (extended discussions are in paragraphs indicated below):

a) Monochord marks for Marchetto's new intervals could be drawn using the same kinds of geometric constructions as had been needed to realize the proportions for earlier medieval (Pythagorean) tuning. Either of 2 propositions from Euclid would suffice—even to realize alternative Renaissance versions of Marchetto's tuning.[2;6]

b):

i) Visually, the marks for Marchetto's truly novel intervals (involving F74, C74, G74, D74) would be quite distinct from those of previous tunings. The intervals in his tuning that involved B or Bb were alternatives to, substitutes for, or re-"ratio"-nalizations of, intervals in previous medieval tuning. These novelties would be hard to distinguish by eye from those they would have replaced, especially in upper registers.[3.0–8]

ii) Sonically, the medieval monochord would be much more accurate than its post-1500 successor. All the same, any sources of measurement error would help persuade listeners of Marchetto's time that differences between his enharmonic/diatonic semitones and their earlier, Pythagorean versions were negligible.[3.9–10]

c) Melodically or in discant, musicians would have little difficulty distinguishing aurally intervals produced by

Marchetto's novel F74, C74, G74, and D74 from intervals of previous medieval practice. Conversely, Marchetto's substitutes for standard Pythagorean notes (B, Bb) generally would pass unnoticed.[4]

d) Marchetto's musical examples would serve not only as an excellent lab demonstration for theorists but also as a superb, step-by-step curriculum for novice singers. His schematic fragments easily could be memorized as exercises and used as a basis for group- or self-instruction.[5]

e) Even if simplistically realized (e.g., as "1/5-tones"), characteristically Marchettan pitch-structures would survive—as they would if the most important mathematical flaw in Marchetto's formulation were removed in a straightforward manner [3.6–7;6]

f) Even if Marchetto's intervals were performed in Pythagorean, equally-tempered, etc. versions, Marchettan structures would persist—albeit to varying degrees—in pieces closest in provenance to his original formulation. Each such version can be understood in its own right—or in fully "de-centred" fashion, as a variant of the others. In principle, fluency in each could be acquired by refining or "de-refining" skills learned for any of the others.[7]

2. GEOMETRIC CONSTRUCTIONS FOR THE MONOCHORD

[2.0] Geometric constructions required for a fastidious, "ideal" realization of Marchetto's tuning on the monochord had been known for about 2 millenia. E.g., Euclid VI,9 (Heath 1926/1956: v.2, 211–12) gave a formulation for dividing any line segment into any number of subsegments having equal lengths. This powerful construction would more than suffice for both Marchetto's dieses and the previously standard medieval tuning. However, both this earlier, Pythagorean tuning and Marchetto's innovative dieses could be constructed entirely by applying Euclid's well known construction (I,10) for bisecting any given line segment (Heath 1926/1956: v.1, 267–68: cf. Adkins 1980).

[2.1] Because the location of the mark for GGG (gamma-ut), the monochord's lowest note, was largely arbitrary (cf., however, [3.0–3.2], below), GGG's sounding length could be established indirectly at the outset by setting BB (a M3 above GGG) at 3 times any feasible length, x, where x = ~1/4 the length available:

```
(bridge)  BB                            (bridge)
<----x----><---------------3x--------------->
```

Merely by cutting off three consecutive segments of length x, BB's effective, sounding string-length (i.e., its distance from the rightmost bridge), would be 3x:

```
(bridge)  BB                            (bridge)
<----x----><----x----><----x----><----x---->
```

Bisecting BB's length once, b, a p8 above BB, would be at (1/2) * 3x = (3/2) x:

```
(bridge)  BB             b              (bridge)
<--(2/2)x-><----(3/2)x----><----(3/2)x---->
<-----------(5/2)x--------><----(3/2)x---->
```

Bisecting BB's length a second time, b1, a p15 above BB, would be at (1/4) * (3x) = (1/2) * (3/2) x = (3/4) x , whereas EE, a p4 above BB (or a p5 below b, or a p12 below b1) would be at (3/4) * 3x = (3/2) * (3/2) x = (3/1) * (3/4) x = (9/4) x :

```
(bridge)  BB      EE       b       b1    (bridge)
<--(4/4)x-><(3/4)x><(3/4)x><(3/4)x><(3/4)x>
<----(7/4)x-------><--------(9/4)x-------->
```

And so forth, downward through the cycle of p5s, for A, D, G, C, F, and Bb.

[2.2] To add Marchetto's new, sharpened notes (e.g., C74), one need only bisect the whole tone above (D/E) twice, and cut off 1 of these 1/4s *below* the lower note (D):

```
|c|                |c74| |d|                 |e|
                         9                   8
                         36                  32
                         36   35   34   33   32
                    37   36                  32
81                  74   72   70   68   66   64
9                        8
```

Because the whole tone (D/E) would form the ratio 9/8 = 72/64, its 1/4s would be formed by marks for 70, 68, 66; the 1/4 below the lower note (D = 72) would be at C74—and C, a whole tone below D, would be at 81.

[2.3] To construct *all* Marchetto's sharpened notes (F74, C74, G74, D74), one could begin the original tuning at f♯ ~1/2-way along the available string, rather than at BB (~1/4 from the bottom) and construct all other marks relative to this f♯. BB's length would be 3/2 times f♯'s; d74's would (37/36) * (9/8) greater; etc.:

```
|d|          |d74|  |e|             (f♯)
                    9                8
                    36               32
                    36  35  34  33   32
               37   36               32
81             74   72  70  68  66   64
9                   8
```

The Pythagorean F♯ (parenthesized) would be much (~ 42%) lower than Marchetto's new F74:

```
               81                          74    72
               |f|                 (f♯)   |f74|  |g|
                                   <--42--><--48-->
                                   <------90------>
```

[2.4] To divide a whole tone (e.g., A/B) into Marchetto's enharmonic semitone (A/Bb77) and diatonic semitone (Bb77/B), one would only have to bisect a whole tone above (B/C♯, where C♯ would be a Pythagorean note not actually used by Marchetto) and cut off one of these 1/2s above the enharmonic semitone's lower note (A):

```
|a|       |bb77|              |b|                 (c♯)
```

		9		(8)
		18	17	(16)
		72	68	(64)
81	77	72		

Because the whole tone above (B/C♯) would be 9/8 = 72/64, its precise 1/2 (i.e., arithmetic mean) would be at 68. Since 72 - 68 = 4, the Bb77 could be marked readily at 77 = 81 - 4.

3. FEASIBILITY OF DRAFTING MARCHETTO'S 9-FOLD DIVISION

[3.0] In Marchetto's period, fundamental frequencies corresponding to notated pitches were not standardized as they are for today's concert musicians (for whom the fundamental frequency of a1 above middle c is ~440–446 Hz). Nonetheless, there were constraints, as always, on vocal ranges of the adults and children who might sing such notes as Marchetto prescribed.

[3.1] Expressed in modern notation, Marchetto's gamut ranged from GGG (gamma-ut) on the lowest line of the bass-clef staff to e1 in the top space of the treble-clef staff. In the late 19th century, Alexander Ellis gave absolute values between 403.9 and 425.2 Hz for various versions of a1 above middle c produced by tuning forks and pitch pipes used in Padua to tune bells and other fixed-frequency instruments (1885/1954: 510). Although his measurements are based on local musical practices during the period 1730–80, i.e., 4 centuries after Marchetto's time, there is little to suggest pitch standards in the Middle Ages diverged much more from modern norms than Ellis's measurements suggest—especially in leading churches, where such fixed-frequency instruments as organs might be played with other instruments or voices, and vocal music called for increasingly large ensemble ranges (cf., e.g., Mendel 1948/1968, especially 167).

[3.2] Additionally, manuscript illustrations, though unreliable for certain details, indicate that monochords of the time were ~3–4 ft. long (cf. the general estimate of 90–122 cm. in Adkins 1980:495). E.g., well-known medieval illustration shows a monochord held by Guido and Bishop Theobaldus (see **Figure 1**), whose adult heights provide rough estimates of the instrument's absolute dimensions (as do the plates in Adkins (1992: v.2, 500–10) and the ~1150 drawing of Boethius (at the SMT homepage; see **Figure 2**).

[3.3] Presuming, at least for the sake of illustration, a monochord whose sounding string was about a yard long, one can estimate quite closely the absolute distances between various marks for the notes it would produce. GGG gamma-ut, corresponding to the lowest line of the modern bass-clef staff could result from sounding a string-length of 36 inches. The highest notes in Marchetto's system that would produce his narrow diesis are e1 and d741 at the modern treble-clef staff's top. Relative to a GGG gamma-ut of 36", their string-lengths would be:

```
     for |e1|, 36*(1/2)*(1/2)*(2/3)*(8/9)     =5.33",
and for |d741|, 5.33*(74/72)                  =5.48".
```

[3.4] Such marks would be about 1/6" apart, i.e., readily distinguishable from each other by instruments used at the time for geometric diagrams. Even if "concert pitch" for such church musicians were fully a p4 higher, one would still be dealing with smallest distances of about 1/8", as one would if a monochord's open string-length were only ~27". An 8ve below, this distance would be twice as great; a 15th below, 4 times as great, i.e., ~.5"—for this, the very smallest interval of Marchetto's formulation.

[3.5] The notes b' and c' (in the middle of the modern treble-clef staff) were the highest for which Marchetto would use his revised, "enharmonic" version of the minor semitone. According to earlier medieval tuning, these notes would be marked off at the following points on a 36" monochord:

```
     for |c1|, 36*(1/2)*(1/2)*(4/3)     =6.75",
 and for |b1|, 6.75*(256/243)           =7.11".
```

Relative to c1 at 6.75", Marchetto's version might be located at:

```
     |b771|, (81/77)*6.75"              =7.101",
 or  |b761|, (76/72)*6.75"              =7.125",
```

i.e., ~1/100" to ~1/70" from the Pythagorean mark, from which neither would be easily distinguished by eye (nor from the other).

[3.6] As a precise calculation, Marchetto's tuning disregards the incommensuracy between 9-fold subdivisions of the spaces from A to B, Bb to C, and B to (Pythagorean) C♯. Overlooked (or ignored) is the mathematical difficulty that 81/77 (in modern decimals, ~1.052) is a smaller ratio than 76/72 (~1.056), but each would be 2 dieses:

```
|a1|      |bb771|        |b1|      (c1)      (c♯1)
81        77             72        68?       64
          81             76        72        68?
(a1)      |bb1|          |b761|    |c1|      (c♯1)
```

Marchetto should have known that for (positive) numbers generally, a/b > (a + x) / (b + x), an important inequality he invoked in discussing the ratios 17/16 and 18/17 (|Luc. 4.11.4|: cf. 72/68 and 68/64, here and [2.6], above; cf. also Euclid V.8 in Heath 1926/1956: v. 2, 149–53).

[3.7] A plausible way out of this difficulty would be to divide the m3 a1/c1 space into 4 + 5 + 4 = 13 dieses:

```
|a1|      |bb3961|       |b3711|    |c1|
32                                  27
<--4*5---><------5*5------><--4*5--->
416       396             371       351
```

The 32/27 m3 ratio could be expanded to 416/351 (by multiplying both numbers by 13, a mathematical "trick" reported by Boethius, e.g., in 5.16.366), so that the 4/9-whole-tone, 2-diesis minor semitones would be 20 = 4 * 5 of the intervening 65 = 13 * 5 = 13 * (32 - 27) parts. Graphically the resulting differences would be only ~1/25" in the highest register, where they could hardly be distinguished by drawing tools of the time—or eyes of any time.

[3.8] In sum, Marchetto's new version of the minor semitone (and hence, its whole-tone complement, the major semitone, e.g., from Bb396 or Bb77 to B371 or B76) produced differences from the earlier medieval values so slight that they could be ignored or exploited persuasively in a visual demonstration, whereas his most striking innovation, greatly raised scale degrees, would be clearly, visibly distinct on the monochord.

[3.9] In addition to problems of drawing and discerning geometric figures, one can assess how vulnerable the sounds of Marchetto's tuning might have been when realized on a necessarily fallible mechanism like the monochord. Although Ellis reported substantial errors in the fundamental frequencies produced on well regarded monochords of his day (1885/1954: 441–42), it should be emphasized that the 3-bridge monochord of Marchetto's period (2 fixed at the ends, 1 movable between—touching the string from below) greatly excelled in accuracy the post-1500 instrument, with fixed bridge and nut with movable tangent between—to press the string to the belly from above. For the latter, Cecil Adkins (1963: 4) reported accuracy of ~0.5 mm.—at e1, ~6.5 cents.

[3.10] Nonetheless, any span of tolerance resulting from changes of tension, friction between bridge and string, aligning bridge and marks by eye, matching by ear pitches on another string (e.g., for Marchetto's dyad examples), variations in temperature and humidity, etc. would add persuasive auditory force to the notion that divergence of Marchetto's enharmonic semitone from the earlier minor semitone was negligible.

4. INTERVAL PERCEPTION AND MARCHETTO'S DIESES

[4.0] In recent music perception experiments I had undertaken quite independently of my Marchetto studies—or so I thought!—I found that for *melodic* (i.e., successive) intervals, my subjects (8 undergraduate music majors) were quite uncertain whether to label a particular melodic interval as, e.g., M3 or m3 if its frequency-ratio was close to 350% (i.e., midway between the ideal values of 300% and 400% for modern, equally tempered, 3- and 4-st intervals). Such uncertainty generally extended from ~330% to ~370%. Moreover, much the same held for other intervals involving notes "in the cracks:" the students generally displayed uncertainty in labeling frequency-ratios within the ranges 130%–170%, 230%–270%, 430%–470%, 530%–570%, . . . 1130%–1170% (see **Figure 3**).

[4.1] These results agreed with other recent studies of the so-called "categorical perception" of intervals (Harnad 1987; Butler 1992: 55; Krumhansl 1991: 281–83). Of greater moment here was an apparent anomaly that arose consistently for each subject. The region of uncertainty in deciding whether to label a melodic interval "unison" or "semitone" appeared not between ~30% and ~70%; instead, from ~15% to ~35%. In other words, a much smaller difference was required to distinguish a semitone from a unison. This is not to say that the students heard all tones differing by less than ~15% as "the same." Post-experiment de-briefing indicated that, if perceived as differing, such tones were heard as not differing enough to constitute a semitone.

[4.2] All students applied with full certainty the label "semitone" to differences on the order of Marchetto's 48%. For differences smaller than ~25%, some offered such responses as "sharp unison." The distinction here seems to have been between 2 distinct pitches, e.g., C♯ and C, and 2 versions of the same pitch, e.g., C♯ and sharp C♯—in other words, between a functional difference and a nuanced sameness. In converse fashion, some offered the idea that smaller versions of intervals heard clearly as semitones were "flat semitones." The following tones illustrate intervals they heard, where 0%=c and 100%=(equally tempered) c♯:

|0%|	|5%|	|10%|	|15%|	|20%|	|25%|
|30%|	|35%|	|40%|	|45%|	|50%|	|55%|
|60%|	|65%|	|70%|	|75%|	|80%|	|85%|
|90%|	|95%|	|100%|			

[4.3] 3 students volunteered the adjectives "sharp" and "flat" for intervals close to each maximally uncertain interval. E.g., at ~340% they volunteered "sharp m3;" ~360%, "flat M3." This tendency to label intervals of ((2n + 1) * 50) +/-10) % sharp or flat recurred in all mid-regions—except for the unison/semitone, where ~20% intervals were labeled "sharp unisons," and ~30% intervals "flat semitones."

[4.4] Marchetto emphasized his new dieses were not for use in chant, rather, in discant, i.e., 2-part *polyphony* and closely related idioms. In the Lucidarium, Marchetto's 27 musical examples of the diesis interval are all cast in a2, note-against-note, "first-species" counterpoint, each illustration presenting a sharpened tone in the 2d of its 3 sonorities.

[4.5] 3 of his musical examples are counter-examples; regular in all other respects, they counter-indicate use of his sharps in oblique motion, specifically against repeated notes that would form with them the following sonority successions (|Luc. 5.6.8-12|):

```
|DDd|_|DDc74|_|DDd|           |CCg|_|CCf74|_|CCg|
p8      M7       p8            p12     x11      p12
            |bd|_|c74d|_|dd|
            m3   m2     p1
```

[4.6] The wide M7 and x11, and especially the narrow m2 he proscribed would produce beating (smooth, continuous fluctuation in loudness: ~ 4–18Hz) or roughness (~18–100+Hz), particularly in lower registers (Stevens and Davis 1938/1983: 242–45). Just below top-space GG in the modern bass-clef staff, a narrow Marchettan m2 (e.g., a dyad, |180-185|, with fundamental frequencies of 180Hz and 185Hz: cf. 74/72= 37/36) would produce beats of 5Hz (i.e., 185-180 = 5 pulses/second: this 'dyad' can help confirm the accuracy of transferred ra files).

[4.7] Well within the range where beating is most prominent (~ 4 to 18 Hz), such effects could be clearly heard and/or felt whenever narrow m2s were produced by a 2-course monochord (or "dichord:" Adkins 1980 and 1991: 33–40, 500–10), or by voices—from top-space GG of the bass-clef staff to e1 at the top of Marchetto's pitch system, i.e., throughout the range where they would occur in Marchettan works if he had not proscribed them.

[4.8] Secondary, "subjective" beating would be especially prominent between the 2d partial (i.e., 1st "overtone") and 1st partial ("fundamental") of tones close to an 8ve apart (i.e., within 4–18 Hz of the 2/1 ratio), if sounded at relatively high intensities (as they would be in vocal music or on monochords/dichords with resonators, post-1200: Stevens and Davis 1938/83: 184–87, 244; Adkins 1980). Throughout Marchetto's pitch system such effects could be heard in the wide M7s he warns against, especially in lower registers:

```
from GGGFF74    cf.  96Hz   vs 2*96*72/74  = ~186.8Hz

to  ed741       cf.  324Hz  vs 2*324*72/74 = ~630.5Hz
```

which yield secondary beat-rates of (respectively):

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(2*96)-186.8  =  5.2Hz (cf. [4.6], above)
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and

```
(2*324)-630.5 =  17.5Hz.
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[4.9] Divergences greater than ~18Hz give rise to pronounced "roughness." Peaking in intensity at ~50 Hz, such roughness would be especially audible in wide M3s of the lowest register, where DDFF74 would produce a roughness rate of 144 - 186.8 = ~ 42.8Hz. Such effects also would be prominent in the lowest p5s (e.g., around AAEE in the bass-clef staff, where 108Hz and 162Hz produce roughness of 162 - 108 = 54Hz) and in narrow m3s above middle c (e.g., around f74a1 in the treble-clef staff, where 432 * (8/9) * (72/74) = ~373.6Hz and 432Hz result in roughness of 432 - 373.6 = ~58.4Hz). By contrast, Marchetto's musical examples tend to locate such potentially problematic intervals well outside their respective regions of maximal roughness: both illustrations of his narrow m3 about an 8ve lower (at FF74a and GG74b, where roughness rates would be ~1/2), and all 7 examples of p5s half an 8ve or more above (1 at EEb, 4 at FFc, and 1 each at ae and bb–f). The only exceptions are 2 of his 3 illustrations of the wide M3 (at EEGG74)—the other appearing at bd74 (i.e., half an 8ve above the interval's roughness peak).

[4.10] In short, one can understand Marchetto's prohibition of coloured m2s and M7s, as well as his tendency to illustrate p5s and coloured M3s (or m3s) in particular registers as a way of preventing—if only in lab demonstrations—salient beats and roughness in discant. Nonetheless, Marchetto (citing Isidore of Seville) tried to realize in his dissonances harsh, unpleasant effects, and his sharpened degrees served as means to such closely controlled ends: specifically, "extreme mixtures of two sounds thoroughly mixed with one another, coming harsh and unpleasant to the ear" (duorum sonorum sibimet permixtorum ad aurem veniens aspera atque iniocunda permixtio: |Luc. 5.2.2|).

[4.11] Except his counter-examples (of wide M7s and x11s, and narrow m2s), all Marchetto's diesis illustrations exemplify sharpened degrees participating in narrow m3s, and wide M3s, M6s, and M10s. Each diverges from its ideal, medieval (or, for that matter, modern, equally tempered) frequency-ratio by about half a semitone (all sizes in cents):

interval	Marchettan	Pythagorean	equal-temp't
m3	252	294	300
M3	450	408	400
M6	948	906	900
M10	1650	1608	1600

[4.12] Although one might expect musicians to respond erratically to Marchetto's m3, M3, M6, and M10, important studies by E. M. Burns and W. D. Ward (1978) and Donald E. Hall and Joan Taylor Hess (1984) argue against such a prediction. Burns and Ward emphasized their subjects were most uncertain (i.e., closest to a 50-50, flip-a-coin response) when asked to label (melodic) frequency-ratios close to 350% as "m3" or "M3." However, several of their formally trained subjects were very certain if asked to isolate intervals "in the cracks" as such—i.e., in contrast to intervals closer to the ideal sizes of 12-st equal temperament. Indeed, the step-like curves for their responses merely shifted ~50%.

[4.13] Hall and Hess asked similar subjects not merely to label simultaneous intervals (with spectra ranging from partials 1–5 to 1–10) but also to characterize each on a 7-point scale of "acceptability." Their results confirm for simultaneous intervals earlier findings concerning category boundaries between melodic intervals a semitone or larger: in general, these appear at (2n - 1) * 50%, where n = 1, 2, 3, . . . (i.e., even for the m2 sonority Marchetto proscribed and that had a ~25% boundary in my study of melodic interval perception: the single outstanding exception in Hall and Hess's study of simultaneous intervals was their high M6/m7 boundary, ~970%).

[4.14] Hall and Hess also emphasized the importance of beats and roughness in describing sonorities, especially if such effects can be traced to the first 5 partials (e.g., C, c, g, c1, e1). Their subjects tended to label "acceptable" intervals close to the following values: 2/1, 3/2, and 4/3 for p8s, p5s, and p4s (as in Marchetto's, and earlier medieval tunings, as well as subsequent formulations of just intonation): 5/4, 5/3, 5/2, and 6/5, 8/5, and 12/5 for, respectively, M3, M6, M10, and m3, m6, and m10, (i.e., the ideal values sought in just intonation). Subjects displayed small ranges of acceptance for perfects, and especially M and m imperfects, in contrast to x4, d5, and all 2s and 7s.

[4.15] Though appearing to support, in particular, just intonation, these data jibe well with Marchetto's diesis tuning. Marchetto's ideal values for perfect intervals would yield no beats or roughness; his coloured intervals (especially his wide M3, M6 and M10, but also his narrow m3, m6, and m10) would fall well within the "unacceptable" characterization of the modern-day musicians Hall and Hess studied. Also important, all intervals in Hall and Hess's study lasted 3 secs.; similarly long durations were notated for simultaneous rough coloured imperfects in music closest in provenance to Marchetto s formulation (and in the dyad examples here).

[4.16] In Marchetto's tuning, "non-coloured" 3, 6 and 10 (e.g., c/e, e/g, e/c , g/e, CC/e, EE/g) diverged quite far from the ideal values Hall and Hess discerned. In Marchettan works, these non-coloured imperfects had rather short durations, as did 2 and 7—especially the (prominently rough or beating) coloured m2, M7, and x11 Marchetto emphatically proscribed. The non-coloured imperfect intervals Marchetto retained from earlier Pythagorean tuning necessarily diverged as much as their precursors from the focal, beat-less, ideal values Hall and Hess found. Moreover, Marchettan substitutes for B♭ and/or B would produce similar divergences (all sizes in cents):

tuning	G/Bb	G/B
1/13s	|291| =(416/396)*(9/8)	|402| =(371/351)*(81/64)
1/9s	|292| =(81/77)*(9/8)	|404| =(76/72)*(81/64)
Pythagorean	|294| =32/27	|408| =81/64
just	|316| =6/5	|386| =5/4

[4.17] In Hall and Hess's study, the relatively narrow m3s (291–300%) and wide M3s (402–411%) would all fall in the "unacceptable" half of their subjects' scaling. The pronounced sensitivity to rather slight divergences from 5/4 etc. Hall and Hess found can be explained by beating, which is especially salient for sonorities of long duration. In actual Marchettan works, the sensitivity Hall and Hess tapped would arise only in such long-held cadence sonorities as |EEcg♯|. Moreover, EEg♯ would be rough irrespective of the beating in cg♯ or EEc, whereas other non-coloured, non-cadential imperfect intervals were much shorter.

[4.18] Recent research on interval perception uniformly shows responses are learned and learnable. Formally trained musicians display much less uncertainty than non-trained (e.g., as measured by better fits to steeper ogives: cf. Figure 3 in [4.0], above);(4) some also sub-categorize reliably in distinguishing among m3, wide m3, narrow M3, etc.; and adapt readily to novel intervals "in the cracks." Cross-cutting these achievements are acoustically and physiologically based, non-cognitive phenomena: beating and roughness. Though these effects can be tapped experimentally by requiring such polar, arguably ethnocentric (or "hodiecentric") responses as "acceptable" vs "unacceptable," they can be channeled stylistically in many ways within particular cultural settings.

[4.19] E.g., in great contrast to Western European ideals of beat-less perfect intervals are the precisely patterned, "shimmering" beat-rates for p1, p8, etc. among the bronze keys and gongs of gamelan; through such carefully crafted timbral structures, professional Indonesian tuners have shaped the individual personalities of entire ensembles—in principle, for centuries (cf. Hood 1960; Susilo 1975; Rahn 1996). By comparison, Marchetto's tuning intensified an earlier medieval opposition between beat-less perfect intervals and all others—especially M3, M6, their inversions, and 8ve-compounds—and provided for vividly sharpened leading tones.

5. PEDAGOGICAL ASPECTS OF MARCHETTO'S FORMULATION

[5.0] As in the Lucidarium, pieces closest in provenance to Marchetto's original account of dieses tend to locate his sharpened notes in the highest voice(s): specifically, from FF74 to d741, in parts designated for boys—in particular, pairs of boys (duo pueri: Vecchi 1954). That boys originally were the main performers of Marchetto's sharps illuminates the reception of his challenging account.

[5.1] Because Marchetto complained his new sharp-sign had been drawn wrongly and, as Karol Berger rightly stresses, his sharps "commonly" (a vulgo: |Luc. 8.1.4, 17|; |Pom., p. [-40-]|) had been called "falsa musica" (lit. false music, in contrast to "color fictitius," lit. fictitious colour: imaginative, in one's head, by ear— |Luc. 2.8.9|; |5.6.27|: cf. Berger 1987: 16), one can conclude his diesis chromaticism had circulated outside his direct purview before 1317–18. Beyond some scores that clearly specify Marchetto's sharp notes (e.g., by a natural, square-B sign with upward stem to the right) and the many Lucidarium copies made, transmission of his tuning must have been largely oral.

[5.2] Marchetto's tuning was absorbed into elementary music instruction (Herlinger 1990). E.g., a rudimentary digest of Marchetto's modal theory, seemingly used to teach neophytes at St. Mark's in Venice, the Ars magistri marchetti (Monterosso 1966), presumed knowledge of his dieses in order to determine whether problematically narrow melodies were authentic or plagal. Plausibly, too, the Hebrew translation of an originally vernacular, Italian digest of Marchetto's modal theory, brought to light by Israel Adler (1971), also referred to dieses after the point where the only surviving copy breaks off. If so, this remarkable work would testify to an unusually wide readership for Marchetto's diesis doctrine.(5)

[5.3] Significantly, too, the extensive compilation of selections from the Lucidarium in ms Vatican, BAV Capp. lat. 206 (ca. 1500: ff. 138–67; cf. Herlinger 1990: 239–40), which seems aimed at more advanced practising musicians, e.g., composers or choral directors (rather than their charges), retains the complex argument Marchetto adduced to support his division of the whole tone—an indication that this apparently speculative material, not readily available outside the Lucidarium, formed the basis of lab demos for the actual mathematics underlying Marchetto's tuning for 200 years (as in the Ancient tradition of Euclid's canon: Mathiesen 1975; cf. Szabo 1978 on the centrality of the canon = monochord = qanun? = qun? for mathematics instruction generally).

[5.4] As well as providing further lab demos for speculative aspects of Marchetto's tuning, the Lucidarium's 25 musical examples of sharps could have introduced novices to his chromatic practice. Among pieces using these sharps, Marchetto's a3 motet Ave, Regina Celorum/ Mater innocencie/[Ite, missa est] (Sanders 1973: 571–73; Gallo 1974; Fischer and Gallo 1985: #37) stands out for the microtonal fluency it presumes of its upper voices at a very early date (see **Example 1**). Conversely, the much later anon. a4 motet Ave, Corpus Sanctum/Gloriosi Stefani/ Protomartiris (see **Example 2**) evidences, in its frequent doublings at the lower 8ve, enduring concern for accuracy in the highest voices (Gallo 1968; Fischer and Gallo 1985: #38).

[5.5] Most of Marchetto's semitone examples are schematic: 17 present the diesis sharp as a chromatic passing tone, e.g.:

```
      ascending:     |FFc|_|EEc74|_DDd|
   or  descending:    |DDd|_|EEc74|_|FFc|
```

thereby illustrating directly Marchetto's chromatic division and providing rudimentary exercises for learning to sing the new intervals. 4 more exemplify the diesis sharp as a chromatic lower neighbour. Of these 2 are censured (cf. [4.6] above):

```
|DDd|_|DDc74|_|DDd|      |CCg|_|CCf74|_|CCg|
```

in contrast to a repeated example of approved usage:

```
|DDd|_|EEc74|__|DDd|
```

[5.6] Of the rest, 6 provide parallel realizations of Marchetto's substitutes for minor and major semitones:

```
|aa|_|GGbb77|_|EEb|     |EEb|_|GGbb77|_|aa|
```

Although Marchetto said direct chromatic progressions between B♭ and B could occur in any kind of music (chant, discant, etc.), he emphasized in the Pomerium (|Pom., [-69-]-[-72-]|) that they were not properly used to form leading tones in cadences.

[5.7] Because Marchettan pieces do not employ such progressions, one can understand his prominent examples of them merely as showing how his tuning would replace earlier versions of B and B♭. Setting such B-B♭progressions in discant thus concretized his distinction between diatonic and chromatic semitones, especially as he tightly juxtaposed their contrasting examples (|Luc. 8.1.3|). His idea that enharmonic and diatonic semitones were not to be used in cadences clarified greatly his concept of cadence and emphasized further that the earlier, Pythagorean versions of imperfect intervals he retained would not draw attention to themselves.

[5.8] Generally, Marchetto presented the simplest, most schematic examples both earlier in a group of 2 to 4, and repeatedly throughout his entire discussion. Such distinctions as between using sharp dieses more or less "properly" (proprie) or "naturally" (naturaliter) are exemplified by changing only a single variable—as in inductive ascent and incremental pedagogy, from Francis Bacon and Johann Heinrich Pestalozzi onward).

[5.9] Later introduced in a few groups of examples are illustrations that presume basic knowledge of his novel division, but set in contexts that extend melodically beyond a whole tone: first in a lower range, plausibly to be sung by older instructors, illustrating the proper, natural, rising resolution: |DDa|_|GG74b|_|aa| thereupon, the less proper, less natural, falling resolution: |EEb|_|FF74a|_|FFc| followed by the proscribed, oblique resolution (cf. [4.6], above: |bd|_|c74d|_|dd| culminating in a non-schematic, but thoroughly idiomatic approach and resolution, entirely within an upper voice: |EEe|_|DDf74|_|CCg| Thus, the principles of Marchetto's tuning could be understood by following his words, observing his monochord marks, listening to, and eventually singing, the accompanying examples: in sum, a cumulative process facilitated by his own cross-referencing of relevant passages in the Lucidarium and Pomerium.

[5.10] Emphasized throughout Marchetto's exposition of diesis tuning were new possibilities for sharp chromaticism, even in monophony. Among Paduan dramatic offices of the time, the Lamentum Beate Marie Virginis (Vecchi 1954: 56–63) realized this possibility amply, gradually unfolding (like other Marchettan works) increasing chromatic complication, before opening into a thorough mixture of unisons and constantly crossing 3ds and 5ths in its binatim-like close.

[5.11] That Marchetto's sharps "properly" participated in one of 2 basic progressions:

```
3–5       cf.  |ac74|_|GGd|
or 6–8    cf.  |af74|_|GGg|
```

(and their inversions) suggests an incipient "chromatic discant modality" that might comprise not only originally modal melodies adapted to polyphony as tenors but also, at least intermittently, unaccompanied melody. E.g., at the end [MIDI] of the Lamentum's main, monophonic section, a concluding discant cadence to D (or G), |EEc74|_|DDd| (or |ac74|_|GGd|), is strongly implied by the melodic progression |c74|_|d|, and intensifies the previous centrality of D, just before the work presses to its |a2| [MIDI] conclusion on G (tuned to equal temperament in its MIDI file).

[5.12] Marchetto carefully delineated genre-based differences in semitone usage. Because he provided the same kinds of musical examples for his initial demonstrations of tuning and dissonance (|Luc. 2.6.4–2.8.9|; |Luc. 5.6.8–27|) as for his innovative account of chromatic permutation, i.e., sharp and flat/natural solmization (|Luc. 8.8.3|), one can surmize he intended all his musical examples to be sung by his readers/pupils only when understood, conceptually and aurally.

[5.13] Since his first groups of examples appeared well before his treatment of sight singing, they must have served first as sounding illustrations on the monochord —or more precisely and plausibly, as all are a2—on 2 monochord or dichord courses tuned in unison. Such an instrumental realization would also make available a constant check on initial attempts to sing Marchetto's sharps against a (generally lower) non-chromatic part. A "tenor" of this sort could be performed on a single course with movable bridge as support for, or challenge to, an upper voice, which could be checked readily by a 2d course with (independently) movable bridge. In this way, Marchetto's pupils could proceed from initial stages of comprehension to fluent vocal application in his more demanding works.

6. OTHER REALIZATIONS OF MARCHETTO'S DIESES

[6.0] Such later writers as Tinctoris (Berger 1987: 22– 29) mention Marchetto's 5-diesis whole tone without mentioning its basis in 1/9-tone division. The space for a whole tone could be divided into 5 equal segments by construing its 9/8 ratio as 45/40. The leading-tone diesis would be 41/40: 42% as compared with 48% for the 1/9-tone ratio 74/72 (=37/36), and similarly perceived as a melodic semitone rather than as a wide unison. In comparison with a string-length of 5.33" for e1 (see [3.3], above), this 1/5-tone diesis's d411 string-length would be ~5.4635": readily visible ~1/7" away from its resolution, but less than 1/50" from its 1/9-tone counterpart (at 5.48"). Respective rates of beating and roughness also would be similar.

[6.1] A 1/5-tone, enharmonic-semitone ratio would be 45/43: ~|79%| as compared with ~|88%| for the 1/9-tone version, and ~|90%| for the earlier, Pythagorean value it would replace. For c1 with string-length 6.75", the 1/5-tone version of b1 would be at 7.06": ~1/20" from the 1/9-tone b1 (at 7.11"), and slightly further from the Pythagorean value (7.125"). Although well within the central range for stepwise melodic semitones (a usual context for mi-fa progressions), B or B♭ tuned this way would produce beats if combined with Pythagorean E or F: above a 162Hz EE, a 1/5-tone b would be ~249.8Hz, in comparison with a 243Hz Pythagorean b, producing 13.6Hz beating an 8ve higher (cf. 486Hz and 499.6Hz).

[6.2] To construct such a 1/5-tone division of, e.g., d/e at d41, one could "back up" even further than for the 1/9-tone tuning: to a Pythagorean g♯ at a little less than 1/4 GGG's length. Regarding this as 4x, bisecting its length twice (i.e., into 4 x-units) would provide e at 5x. Bisecting e's length thrice (into 8 divisions of .625x) would result in d at 5.625x (= (9/8) * 5x). Bisecting the space between the g♯ and e thrice, into 8 divisions of .125x, would give d41 at 5.125x (= (41/40) * 5x), etc.:

```
|d|            |d41| |e|                          (g♯)
45              41    40                           32
                                                   4x-->

                      5x-->

5.625x-->

                5.125x-->
```

[6.3] As a plausible solution to incommensuracy problems for B and B♭ in any such tuning, the space for the Pythagorean m3 A/C (32/27) could be construed as 7 (= 2 + 3 + 2) "1/5-tone" dieses. A/C divided into 7 equal segments would produce 224/214 (= 112/107 = 80%) for A/Bb and 199/189 (= 89%) for B/C: cf. 90% for Pythagorean B/C. Combined with Pythagorean values, this 1/7-m3 tuning would produce virtually beat-less realizations of E/B, but Bb/F would beat as in 1/9-tone tuning. Because none of these p5s would appear in a Marchettan cadence (since no sharp leading tone, i.e., sharp E or A, was available for them to resolve—though 1/7-m3 B would support 1/5-tone D41 in a cadence to A/E: BD41_AE)—such effects would pass unnoticed, like those of their uncoloured imperfect counterparts.

[6.4] Among copies of the Lucidarium there was great inconsistency in notating Marchetto's dieses. Even more difficult to assess are pieces that originally might have been composed and/or performed with Marchetto's chromaticisms but that survive only in copies lacking his explicit signs. Determining intonation for such pieces is all the more difficult because of the continuing controversy and confusion his doctrine provoked and wide variation in successors' usage of Boethian semitone terms he adopted. (5)

[6.5] Among later tunings, the recurrent 4-dieses-plus-comma formulation seems quite parallel to Marchetto's 5-fold partition of 1/9-tones, especially as his odd-number doctrine would identify the comma (or diacisma) with his middlemost diesis, 77/76 (= ~23%—cf. the Pythagorean comma: 24%). Moreover, terse references to such seemingly non-Marchettan whole-tone divisions as into 4 or 8 parts (e.g., in Tinctoris 1475/1963) might merely record widespread tuning mnemonics for 9- or 5-fold division via adjacent whole tones: 72/64 or 40/36. Nonetheless, the possibility remains that works originally conceived in 1/9- or 1/5-tone tuning were actually sung with other, e.g., Pythagorean, intervals.

[6.6] If all Marchetto's sharps were rendered in Pythagorean tuning—or as today, in equal temperament— only Marchetto's leading tones would be affected greatly. The extremely wide, coloured intervals could assume the Pythagorean sizes and qualities of other early idioms; p8s, p5s etc. would sound and be performed much the same as in 1/9- and 1/5-tone tunings; not highlighted in cadences, non-coloured imperfect intervals and dissonances would pass as little noticed as in a 1/9- or 1/5-tone performance: in metrically weak positions, for short durations, or as additions to more salient structures.

[6.7] Prominent in Marchettan a3 and a4 works were such progressions as |EEcg♯|_|DDda1|. Arguably, 1/9- or 1/5-tone tuning alone would highlight physiologically and cognitively the core, contrary-motion, leading-tone, discant structure of such arresting cadences: EEg♯_DDa1. Relegated to a subsidiary structural role would be the (much) augmented 5's similar motion to p5: cg♯_da1, c being construed also as forming a structurally less salient, but beating, m6 with EE, with which it would proceed stepwise in non-cadential contrary motion to d: EEc_DDd.

[6.8] In listening to a modern, equal-temperament rendering of Ave, Regina Celorum/ Mater innocencie (audio for Ave, Regina Celorum [MIDI]), a work in which this a3 progression forms the initial cadence, one can merely imagine, "fictitiously," as it were, the effect produced if the already prominent cadential sharps were realized only ~48% (or ~42%) from their resolutions.

7. MARCHETTO'S CHROMATICISMS AND RECENT SCALE THEORY

[7.0] Reversing such an exercise, one can consider the effects Marchetto's tuning would have on the diatonic collection as understood of late. In such a view, one construes sharpened and flattened forms of degrees as replacing, at least temporarily, their natural counterparts. As well, one specifies the structural changes that take place when one or more degrees are altered in various ways (cf. Rahn 1991: 35–44; Clough and Douthett 1991: 125–44).

[7.1] Of particular concern here are contradictions and ambiguities. E.g., if the "white-key" collection is understood as 12 equally tempered semitones, it is remarkable for comprising no contradictions and only 1 ambiguity, namely, between its x4 (FB) and its d5 (BF), where intervals of differing degree-sizes (4th, 5th) have the same sizes in cents or semitones: 600% or 6 st. In a Pythagorean construal, there are no ambiguities, but the FB/BF pair forms a contradiction: an interval of smaller degree-size (4th) has a larger frequency-ratio (612%) than an interval of larger degree-size (5th: 588%).

[7.2] In Marchettan works, sharps generally appear 1 at a time: e.g., C♯ returns to, is "re-replaced" by, C before F♯, G♯, D♯, or B♭ replaces F, G, D, or B—in contrast to later use of 2 sharpened degrees in "double-leading-tone" cadences: e.g., EEGG♯c♯_DDad. The following figure compares consequences of replacing B by B♭, F by F♯, C by C♯, G by G♯, and D by D♯ in 2 frameworks:

a) equally tempered 12-semitone (cf. modern, e.g., keyboard, realizations);

b) Pythagorean tuning (cf. historical reconstructions of early music):

altered degree:	intervals affected:	ideal tuning (in cents): a) equal temp't	b) Pythag'n
Bb	BbE/EBb	600/600	612/588
F♯	CF♯/F♯C	600/600	612/588
C♯	GC♯,FB/C♯G,BF	600/600	612/588
	C♯F/FA,GB,AC♯	400/400	384/408
G♯	DG♯,FB/G♯D,BF	600/600	612/588
	G♯C/CE,EG♯,FA	400/400	384/408
	FG♯/DF,G♯B,AC,BD	300/300	318/294
D♯	AD♯,FB/D♯A,BF	600/600	612/588
	D♯G/CE,FA,GB,BD♯	400/400	384/408
	CD♯/EG,AC,BD/*D♯F*	*300*/300/*200*	*318*/294/*180*

D♯F/FG,GA,AB	200/200	180/204

[7.3] As the above figure shows:

i) replacing C by C♯ (cf. "ascending melodic minor") yields 12-st ambiguities or Pythagorean contradictions between the d4 C♯F and the M3s AC♯, FA, and GB;

ii) a corresponding result obtains for the d4 G♯C if G is replaced by G♯ (cf. "harmonic minor"); also the x2 FG♯ produces ambiguities/contradictions with the m3s DF, G♯B, AC, and BD.

iii) substituting D♯ for D results not only in further ambiguities/contradictions (between the d3 D♯F and the M2s FG, GA, and AB), but also a 12-st contradiction between the D♯F d3 (200%) and the CD♯ x2 (300%), and for a Pythagorean construal, a "doubly contradictory" relation: not only is the CD♯ x2 (318%) larger than the m3s EG, AC, and BD (294%), but also both sorts are larger than the M2s FG, GA, AB (204%), which are larger than the d3 D♯F (180%)!

[7.4] Whether Marchetto's chromaticisms are realized by modern equal temperament or historical Pythagorean tuning, four gradations can be acknowledged:

a) substituting F♯ for F (or B♭ for B) complicates an originally diatonic collection merely by virtue of adding a new pitch class to the piece or passage as a whole; otherwise, the new collection created by the substitution has the same profile of ambiguities or contradictions as the diatonic original—the tritone merely moves to another pair of scale degrees;

b) replacing C by C♯ adds a further tritone pair as well as ambiguities/contradictions around the d4 C♯F;

c) whereas all such complications appear if G♯ replaces G, ambiguities/contradictions around the x2 FG♯ are also introduced;

d) finally, if D♯ replaces D, the same kinds of ambiguities/contradictions arise: additionally, there are ambiguities/contradictions around the D♯F d3, and most important, whether the framework be equally tempered or Pythagorean, this interval not only contradicts the larger x2 CD♯, but does so doubly. In short, even without adopting Marchetto's tuning, chromaticisms of these kinds produce a coherent, gradated increase in complication: from a simple, diatonic starting-point to the complications of D♯.

[7.5] Taken at face value or understood as an approximation to quartertone equal temperament, Marchetto's intervals intensify complications found already in simpler tunings. As the following figure shows, from F♯ onward all the ambiguities in an equally tempered, 12-st construal become contradictions, whether the framework comprises 24 quartertones or Marchetto's reconfiguration of Pythagorean values. Of these possible construals, Marchetto's tuning produces greater contrasts between contradictory intervals, reaching a climax (or crisis) at the d3 D♯F (138%), which is fully 222% smaller than the x2 CD♯ (360%), which is, to re-work a once-popular song title, "its own scale-degree construal's frequency-ratio grandparent." If Marchetto's diesis-based versions of B and B♭ are incorporated, these contrasts increase, but only slightly (on the order of ~2–6%):

altered degree:	intervals affected:	ideal tuning (in cents): equal temperament (quartertone)	Marchettan (1/9-tone)
Bb	BbE/EBb	600/600	612/588
F♯+	CF♯+	650	654

	F♯+C	550	546
C♯+	GC♯+;FB	650;600	654;612
	BF;C♯+G	600;550	588;546
	AC♯+;FA,GB	450;400	450;408
	C♯+F	350	342
G♯+	DG♯+;FB	650;600	654;612
	BF;G♯+D	600;550	588;546
	EG♯+;CE,FA	450;400	450;408
	G♯+C	350	342
	FG♯+	350	360
	DF,AC,BD;G♯+B	300;250	294;252
D♯+	AD♯+;FB	650;600	654;612
	BF;D♯+A	600;550	588;546
	BD♯+;CE,FA,GB	450;400	450;408
	D♯+G	350	342
	CD♯+	*350*	*360*
	EG,AC,BD;*D♯+F*	300;*150*	294;*138*
	FG,GA,AB	200	204
	D♯+F	150	138

[7.6] All differences effected by adopting narrow semitones merely intensify complications that would arise in earlier Pythagorean, or modern equally tempered, 12-semitone or 24-quartertone construals. Moreover, in each of these, the divergent tones form a coherent grouping of their own, parallel to the diatonic originals—a "displaced cycle," as it were, shadowing the cycle of 5ths. F♯, C♯, G♯, and D♯ form, among themselves, a secondary cycle of 702% or 700%, at distances of 100 (= 600 - 500)%, 114 (= 612 - 498) %, 150 (= 650 - 500) %, or 156 (= 654 - 498) % above F, C, G, and D. For 2/9-tone dieses, these cycles are as follow:[(6)]

```
    F74 <--702%--> C74 <--702%--> G74 <--702%--> D74
    /              /              /              /
  156%          156%          156%          156%
  /             /              /             /
 F  <--702%--> C  <--702%-->  G  <--702%-->   D
```

[7.7] If Marchetto's single-diesis semitones are heard or performed as nuanced versions of more usual semitones, i.e., not merely as "semitones" nor as full-fledged "quartertone" intervals, but as "narrow, small, or sharp semitones," the nuances that result can be construed as forming similarity relations among themselves, e.g., narrow, small, or sharp "to the same extent" or "by the same amount." In this way, putatively quantitative divergences can be understood as proportionally qualitative or qualified—as it were, "adverbially" (e.g., DF74 is smaller than DG "by as much as" F74A exceeds GA), rather than “adjectivally” or as “nouns” in their own right (e.g., DF74 and F74A are a “large” M3 and a “small” m3, or “a p4 minus a diesis” and “a M2 plus a diesis”): put another way, not as separate, distinct “kinds” of intervals nor merely as “marked” intervals, but as intervals aletered or varied in a shared, common way and forming a cycle of their own.

[7.8] That a wide M3 would be understood as a version “of” a diatonic M3, rather than vice versa—and rather than each being construed as “allophonic” or “in free variation” with the other (cf. allophones or phonetic variations within a single phoneme)—is assured by the consequences: CE and GB match each other within a passage where F♯+ is the only chromatic note; FA and GB match each other within a passage where C♯+ is the only chromatic note, whereas DF♯+ and AC♯+ match each other only between such passages; however, CE and FA match each other across such passages also; instances of GB match within and between such passages; etc.

[7.9] Because Marchettan chromatic intervals idiomatically are always “out-numbered” by their diatonic counterparts, they are sites of complexity. Disadvantaged by their opposition to the many matching relations among other intervals of the same scale-degree size, each chromatic interval, on its own, would be rather difficult to learn (as are the similarly rare tritones within the diatonic collection). However, that they share extents by which they diverge from their majorities (e.g., AC♯+ and C♯+E vs FA, GB, and DF, EG, BF, etc.) provides a starting-point for a “progressive” Marchettan pedagogy, with direct extensions available through matching across time-spans (e.g., AC♯+, EG♯+).

[7.10] Cross-cutting the simple-to-complex ordering of chromatic effects from F♯ to D♯ are the potential reinforcers served up by the observation that each sharp forms with several of its diatonic passage-mates similar intervals as the others (cf. F♯+ vs G, C, D, and D♯+ vs E, A, B). Rather than shaping each sharp’s intonation merely with its resolution (e.g., by carefully tuning F♯+ relative to G, as in chromatic neighbour-tone figures), or construing its intonation merely as diverging from a referential value (e.g., by tuning F♯+ relative to F, as in a chromatic passing-tone figure), or attending only to such connections within an initial stage (e.g., F♯+ along with C♯+, G♯+, and D♯+, and/or F♯+/G along with C♯+/D, G♯+/A, and D♯+/E), Marchettan sharp-structures suggest a richer curriculum that would introduce non-leading-tone/ altered-tone successions early on: D/F♯+, A/C♯+, C/F♯+, etc.), and amplify such melodic tasks with discant—much as the Lucidarium’s musical examples imply and Marchettan works require.

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Footnotes

1. References to the Lucidarium follow Herlinger's numbering of its tractata, chapters, and clauses (1985). The | sign indicates links to the edition at the TML website http://www.music.indiana.edu/tml/14th. [EDITORIAL NOTE: The URL leads to fourteenth-century texts at the TML site., which has both the Gerbert version (MARLU) and the Herlinger edition (MARLUC). Each *Ludicarium* tratctatus is in a separate HTML file, in the form MARLUC#_TEXT.html, where # is the number of an individual tractatus in the treatise (e.g. MARLUC8_TEXT.html).]
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2. Below, a tone designated by letter-name within | signs, e.g., |c|, can be heard, in the html version, by clicking on the

letter-name, or getting the corresponding RealAudio (ra) file, e.g., http://mto/audio/4.6/c.ra. [**Editorial Note:** Users may need to adjust the volume when playing some audio files.] For sound files, bass-clef staff letter-names are capitalized and tripled (GGG for 1st-line), or doubled (AA, BB, . . . GG for 1st space to top space). Upper registers are designated by single letters (from a, just below middle c to g—with a numeral 1 for an 8ve higher: a1, b1, . . .). By contrast, single capitals (A, B, . . .G) designate any member of a pitch class. Flats are designated by b (e.g., Bb for B-flat); Marchetto's high sharps, by their characteristic numbers (e.g., C74); non-Marchettan sharps, by # (e.g., C#). In the sonic examples, c is middle c (256Hz); unless otherwise indicated, tuning is Pythagorean (e.g., g = (3/2) * 256 = 384Hz).
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3. Oliver B. Ellsworth (1987: 340) gives a clear account of how Marchetto adapted earlier semitone terms by shifting each "up 1 notch."
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4. In my study of melodic interval perception, all ogives (cf. Guilford 1954) were significantly close to (i.e., diverged non-significantly from) the students' responses at the p <= .05 level in a standard chi-squared test for grouped data (on which see Smith 1985: 319–414). For intervals of 1 to 12 semitones, responses were grouped into 5-cent increments within the medial, (100n +/-30)%) range; for the unison/semitone pair, within the (5 45)% range.
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5. Karol Berger (1987: 16–29) comprehensively surveys successors' responses to Marchetto's terms and notational signs. As Berger (26) and Ellsworth (1987: 337–38) emphasize, the Lucidarium survives in 15 full and 3 incomplete copies (~1317–1500), and the Marchettan orthodoxy of Bonaventura da Brescia's Breviloquium enjoyed 19 early editions (1497–1570).
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6. Mieczyslaw Kolinski's Pythagorean formulation of the 22 srutis of Ancient South Asian tuning (1961) would also result in a displaced cycle and satisfy the scale analysis of Clough et al. (1993), the latter re-framed as comprising 2 kinds of intervals: a = 90% and b = 24%. In such a construal, a sruti could be understood, like Marchetto's diesis, as the difference between consecutive strings, marks, or frets on a tuning instrument (e.g., of the vina variety).
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