## MTO 9.1 Examples: Honing, Some Comments on the Relation Between Music and Motion

(Note: audio, video, and other interactive examples are only available online) http://www.mtosmt.org/issues/mto.03.9.1/mto.03.9.1.honing.php

## **Example 1.** Formal models of the 'final ritard' in music performance

## Formalizations of the final ritard in music performance (see bibliography for references)

Below some of the existing formalizations of the final ritard are summarized. Kronman & Sundberg (1987) define the final ritard as a square root of score position, a model of constant braking force (a convex function):

$$v(x) = (u^2 + 2ax)^{1/2} \tag{1a}$$

Longuet-Higgins & Lisle (1989) and Todd (1992) propose an identical model, but express it as tempo linear in time:

$$v(t) = u + at \tag{1b}$$

Friberg & Sundberg (1999) generalize this model by adding a variable q for curvature (varying from linear to convex shapes), w (a non-zero final tempo), and normalize it:

$$v(x) = [1 + (w^{q} + 1)x]^{1/q}$$
(2)

Todd (1985) and Repp (1995) suggest quadratic IOI (or beat duration) as a function of score position:

$$IOI(x) = c + kx + lx^2$$
(3)

This results in a concave function when expressed as tempo as a function of score position. In addition, Feldman, Epstein & Richards (1992) and Epstein (1994) discuss a model of force dynamics. However, they tested it with an unrelated model of beat duration, similar to Eq. 3 (cf. Friberg & Sundberg, 1999).

## List of symbols

- a acceleration
- c constant, reflecting vertical displacement
- IOI inter-onset interval (or beat duration)
- k, I coefficients reflecting degree of curvature
- q constant, identifying curvature
- t time
- u initial tempo
- v velocity (or tempo)
- w final tempor
- x distance (or score position)

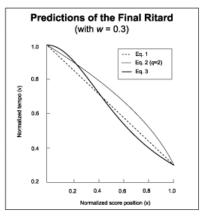


Figure 1. Kinematic models of the final ritard in music performance.

**Example 2.** A mechanical implementation of a contast braking force model, consisting of a music box (1), a piece of piano roll (2), solid-metal flywheel (3), belt (4), and a handle (5).

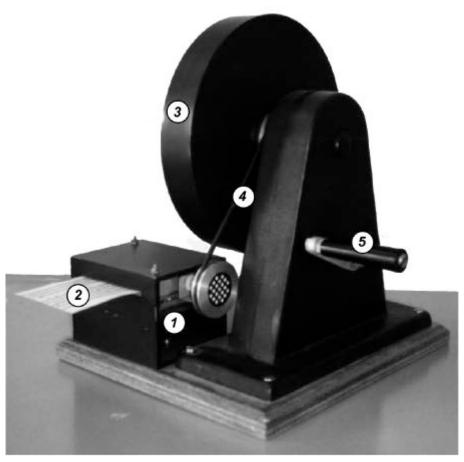


Figure 2. A mechanical implementation of a constant braking force model, consisting of a music box (1), a piece of piano roll (2), solid-metal flywheel (3), belt (4), and a handle (5).

**Example 4.** Final ritards in performance of the last three measures of R. Schumann's *Träumerei* 

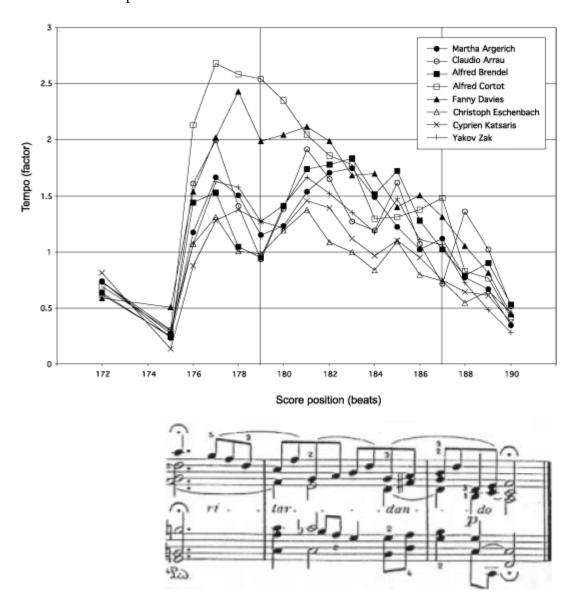


Figure 3. Final ritards in performances of the last three measures of R. Schumannís Traumerei (selection from Repp, 1992)(See footnote 3 in article)(Tempo 1 equals M.M. eighth-note=60.)