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A Review of the 2003 American Mathematical Society Spring Southeastern Section Conference, Special Session on Mathematical Techniques in Musical Analysis

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ABSTRACT: This article consists of a review of the nineteen presentations in the 2003 American Mathematical Society Spring Southeastern Section Conference, Special Session on Mathematical Techniques in Musical Analysis.

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[1] The American Mathematical Society's 2003 Spring Southeastern Section Conference was held Friday–Sunday, March 14–16, on the campus of Louisiana State University in Baton Rouge. It included a special session, "Mathematical Techniques in Musical Analysis," which featured the work of twenty music theorists and mathematicians. The session organizers were Robert Peck, Assistant Professor of Music Theory at Louisiana State University; and Judith Baxter, Director of the Liberal Arts and Sciences Academic Advising Center and member of the faculty of the Department of Mathematics, Statistics, and Computer Science at the University of Illinois at Chicago. The entire special session was divided into four presentation sessions: Friday afternoon, Saturday morning, Saturday afternoon, and Sunday morning.

[2] The first presentation session opened with remarks by Ronald Ross, Dean of the College of Music and Dramatic Arts at Louisiana State University. Dean Ross welcomed the presenters and other conference attendees, and announced the acceptance of funding for the formation of LSU CAPITAL, a center for interdisciplinary studies of the arts and digital technology at Louisiana State University. Following Dean Ross, Robert Peck (rpeck@lsu.edu) gave the first presentation of the special session, "Introduction to the Special Session: An Overview of Mathematical Techniques in Musical Analysis." Peck provided a general historical outline of mathematical techniques in musical analysis, concentrating primarily on those aspects of acoustics, set theory, and group theory that would be amplified in later presentations.

[3] The next speaker was Stephen Soderberg (ssod@loc.gov) of the Library of Congress. Soderberg's paper, "The Impossible Coin Toss," presented the problem of finding a unified solution to the Z-related set problem. He traced the history of the theory of Z-relations, citing Milton Babbitt's Generalized Hexachord Theorem (GHT) and David Lewin's Z-related triples in modulo18. He discussed Z-relations further in terms of his own dual inversions, which can account for Lewin's modulo 18 triples, as well as other n-tuples in spaces of even cardinalities. However, he pointed out that neither the GHT nor the dual inversion solution addresses all instances of Z-relatedness.

[4] Jack Douthett (jdouthett@tvi.cc.nm.us), of TVI Community College, spoke next, about the concept of maximal evenness. His paper, "Dinner Tables, the Ising Model, and the Piano Keyboard," presented the concept of maximal evenness in terms of guests sitting around a table at a dinner party. Specifically, Douthett asked how one might seat a certain number of French and American guests, so that each guest might enjoy conversation with neighbors from the other country. He then presented the question as a general mathematical problem, and pointed out how the solution was arrived at first as a music theoretical result in scale theory. He drew further analogies to the one-dimensional Ising model of physics, in which a maximally even distribution of upward and downward spins minimizes the overall energy, and to the piano keyboard's distribution of white and black keys.

[5] The first presentation session concluded with the work of two graduate students. The first was Panayotis Mavromatis (pm@theory.esm.rochester.edu) of the Eastman School of Music. Mavromatis's presentation, "Minimal Description Length: An Information-Theoretic Approach to Music Model Building," provided an approach to oral-based musical traditions, for which no explicit documentation of rule systems exists or is known. These rule systems may involve such activities as composition, improvisation, or listening. The particular repertoire he considered is modern Greek church chant. Drawing on statistical techniques, he built a stochastic model of melody in this corpus. Specifically, he defined a Hidden Markov Model (HMM), using a variant of Stolcke and Omohundro's state merging algorithm, with Rissanen's Minimal Description Length (MDL) as the termination criterion. Among the questions raised after the talk was how this model might be used to analyze jazz improvisation.

[6] The final speaker in the first presentation session was Jonathan Wild (wild@fas.harvard.edu), a graduate student at Harvard University. His talk, "Tessellating the Chromatic," dealt with concepts related to combinatoriality as applied to pitch-space, rather than pitch-class space. For example, he examined how one might successively overlap members of one particular set-class, thereby packing, or tessellating the entire chromatic. He presented various types of pitch tessellation, drawing on the work of Coven and Meyerowitz. In terms of a compositional technique, which also has implications for analysis, he also examined Z-related tiles, simultaneous tilings of pitch and rhythm, and issues raised by other (non-twelve) equal-tempered spaces.

[7] The second presentation session opened with a talk by Thomas Noll (noll@cs.tu-berlin.de), of the Technical University Berlin. His presentation was titled "A Mathematical Model for Tone Apperception." It applied mathematical concepts to apperception psychology, as a means of modeling musical ambiguity. Drawing on ideas of Wilhelm Wundt (such as Wundt's eye metaphor), Noll described apperception in terms of inner vision, wherein ideas enter the scope of attention either passively or actively. This process may be modeled via a symplectic geometry. It has implications for Riemann's musical motion metaphors, Lewin's GIS-model, Gollin's transformational approach to enharmonicity, and Meeus' Neo-Rameauean approach to tonal progressions. Specifically, Noll invoked the discrete subgroup of integral symplectic matrices $SL_2(Z)$, wherein the upper triangle matrix corresponds to the "passive fifth step" (e.g., I/I -> V/I), while the lower triangle matrix corresponds to the "active fifth shift" (e.g., I/I -> I/V).

[8] The second presenter was Adrian Childs (apchilds@arches.uga.edu) of the University of Georgia. In his presentation, "Structural and Transformational Properties of All-Interval Tetrachords," he spoke about the all-interval tetrachords (4-Z15 and 4-Z29) not in terms of their Z-relatedness, but rather in terms of their all-interval construction. He went on to define a common-tone transformational scheme that results from the partitioning of the tetrachords over the set of interval-classes. He demonstrated further how certain tonal and octatonic properties of these tetrachords may function as a means of generating musical growth processes, using excerpts from his own composition *Shadows Numberless*.

[9] Next was a presentation by Clifton Callender (callen_c@cmr.fsu.edu) of Florida State University. His talk was titled "Continuous Functions in Musical Spaces." Instead of dealing with finite quotients of the familiar pitch-, time-, and tempo-continua (equally tempered pitch-space, even pulse-space, etc.), his presentation involved the infinite spectra of such domains. First discussing pitch, he provided various analytic perspectives of Kaija Saariaho's *Vers le blanc* (IRCAM, 1982). He went on to demonstrate how similar techniques could be applied to trajectories in modular rhythm and tempo set-class spaces, using Nancarrow's Study No. 22 and his own Canon as examples of the latter techniques.

[10] The second presentation session concluded with two talks on scale theory. David Clampitt (david.clampitt@yale.edu) of Yale University gave the first, "Number Theory and Music Theory: Recent Extensions in Music Theory of the Three Gap Theorem." His presentation examined the implications of the Three Gap Theorem in music scale theory. Specifically, the Three Gap Theorem (proven by V. T. Sós in 1958) states that, for a finite set of consecutive integer multiples of a real number modulo 1, the "gaps" or distances between successive elements come in at most three sizes. Several familiar music

scalar constructions display this property. For example, following a particular proof of Myhill's Property, Clampitt demonstrated how the white-key diatonic and the black-key pentatonic collections are both examples of scales in which generic steps come in just two sizes.

[11] The last presentation in this session was by Norman Carey (ncarey@esm.rochester.edu) of the Eastman School of Music (Jeffrey Perry of Louisiana State University read Carey's paper, as Carey could not be present). Carey's talk, "The Coherence Index in Well-Formed and Pair-Wise Well-Formed Scales," furthered certain ideas introduced by Clampitt in the previous presentation. Specifically, Carey introduced the notion of "coherence" in well-formed and pair-wise well-formed scales. Such coherence is measured in terms of a Coherence Index: for any set of N elements modulo 1, the set is coherent if, for all n, all intervals of span n are smaller than any interval of span n+1. The result Carey drew is that well-formed scales are particularly more coherent than the average collection, and he demonstrated the specific ways in which incoherence is limited in pair-wise well-formed scales.

[12] The third presentation session opened with a talk by Richard Cohn (r-cohn@uchicago.edu) of the University of Chicago. Cohn's presentation, "A Tetrahedral Model of Tetrachordal Voice-Leading Space," replaced his scheduled talk on LPR sentences. It consisted primarily of new work in progress, for which Cohn solicited feedback from the audience. Specifically, departing from earlier work by Joseph Straus and Ian Quinn, Cohn constructed a five-level tetrahedral graph of various parsimonious and transpositionally combinatorial relations among the set of all tetrachordal set-classes. The base of this pyramidal graph and its two subsequent lower levels model P2 relations, while adjacent links between these levels incorporate P1 relations. The upper levels and their links display transpositional combination shadows and metashadows. The tetrahedron, as viewed from various perspectives, demonstrates certain symmetries among the set of tetrachordal set-classes.

[13] The second speaker was Jonathan Kochavi (kochavi@acsu.buffalo.edu) of the University of Buffalo. (Edward Gollin, who was scheduled to present next in this session, cancelled his talk, as he was unable to attend.) Kochavi's presentation was titled "An Algebraic Classification of Contextually Defined Musical Inversions." In it, he generalized the concept of contextual inversion in terms of the three parsimonious neo-Riemannian operators. Then, adjoining these contextual inversions to normal transposition, he defined various transformation groups, all of order twenty-four. Finally, he examined the various algebraic structures among these transformation groups, and presented musical contexts for each.

[14] Next, Julian Hook (juhook@indiana.edu) of Penn State University (now of Indiana University) gave a presentation, "Uniform Triadic Transformations: A Wreath Product in Music Theory." Hook's talk dealt with a group of 288 uniform transformations on consonant triads. Specifically, a uniform triadic transformation (UTT) is represented by an ordered triple <σ,t+,t->, in which σ represents either the preservation of, or change of, a triad's mode; t+ represents its transposition on major triads; and t- represents its transposition on minor triads. The uniformity condition states that a UTT transforms all major triads consistently, and also transforms all minor triads in a consistent way. He demonstrated how the familiar Riemannian group of schritts and wechsels forms a subgroup of this transformation group, pointing out how the UTTs reflect their characteristic Riemannian dualism. In addition to other simply transitive subgroups, he also defined a larger group Q that contains the familiar inversion operations, transformations on non-triadic set-classes, and some diatonic and serial applications.

[15] The third presentation session concluded with a talk by Michael Buchler (michael.buchler@music.fsu.edu) of Florida State University. His presentation was titled "Notions of Equivalence and Similarity in Atonal Music Theory." He started by describing how, in the absence of a system such as tonality, composers in the twentieth century turned to other means of projecting coherence and continuity in their music. As a result, music theorists have devised various means of measuring notions of equivalence and similarity in music. He provided a general survey of these types of measurements, focusing largely on their emphasis on either object-driven similarity or transformation-driven similarity.

[16] The last presentation session began with the work of two graduate students at the Eastman School of Music. The first was Richard Randall (randall@theory.esm.rochester.edu), whose presentation was titled "Music, Models, and the Relative Complexity of Analysis." Departing from Casti, he described the complexity of a musical analysis as the degree of interaction between the musical objects being analyzed and the particular analytical methodology employed. He defined analysis as an ordered triple A = <S,M,Z>, where S is a set of objects to be analyzed, M is the analytical methodology, and Z is the set of states that characterize S in terms of M. He went on to define comparative analysis of degrees of interaction as an ordered triple C = <S,M,Z>, where now S is a set of A's as above.

[17] The second presenter in this session was Ian Quinn (iquinn@uchicago.edu), who is completing a visiting position at the

University of Chicago (and joining the University of Oregon). Quinn's presentation, "Chord Quality and General Harmony," presented many of the concepts of pitch-class set theory and transformation theory as discrete Fourier transforms. In particular, he defined a group of operators that includes transposition (T), inversion (I), multiplication by 5 modulo 12 (M), and pcset complementation (C). Then, he presented a planar graph generated by the above transforms, in which nodes represent pcsets, and edges represent parsimonious and quasi-parsimonious voice-leading motions between pcsets. He defined various measurements in this plane, including a "lewin," which serves as the standard length.

[18] After a break for one of the conference's keynote addresses (Günther Lumer of the University of Mons-Hainaut), the final presentation session of the special session concluded with three talks. The first was by Eytan Agmon (agmone@mail.biu.ac.il) of Bar-Ilan University. Agmon's presentation, "Numbers and the Western Tone-System," departed from concepts in psychoacoustics. Specifically, he noted that we use a logarithmic function of frequency to measure (equally tempered) pitch, and that certain ratios of frequencies, particularly simpler ratios, have special auditory status. He noted how these psychoacoustical properties do not always correlate to Western notions of musical space. That is, while such acoustic measures function as real numbers, much of our experience with music is in terms of integers. Musical pitches can be so modeled in terms of chromatic space (specific) and diatonic space (generic). To that end, he devised a triple, (C,p,rate(p)), in which C is a core pitch as a specific/generic integer pair, p is any real number, and rate(p) is a measurement of the "in-tune-ness" of p to C.

[19] The next presentation was by John Rahn (jrahn@u.washington.edu) of the University of Washington. Rahn's talk was titled "Some Recent Developments in Mathematics Applied to Music Theory." Rahn presented summaries of two large and significant contributions to the field of mathematics and the arts. The first book he discussed was Guerino Mazzola's *The Topos of Music* (1999). Drawing on Mazzola's earlier work, this book puts forth a classification theory of musical objects. It contains further topologies for various musical domains, including melody, rhythm, and harmony, especially in the contexts of counterpoint and modulation. It also deals significantly with a theory of musical performance; using Lie algebra, it extends its concepts to object-oriented software environments. The second book Rahn discussed was Michael Leyton's *A Generative Theory of Shape* (2001). Dealing not only with music but with the arts in general, Leyton's book presents a theory of (complex) shape with regard to two properties of intelligence: transfer of structure and recoverability of the generative operations, with the ultimate goal of deriving understandability from complexity. To this end, Leyton defines a class of unfolding groups, which "unfold" complex shapes from their maximally collapsed versions.

[20] The final presentation was of work by Ciro Scotto (cscotto@esm.rochester.edu) and Robert Morris (mris@mail.rochester.edu), both of the Eastman School of Music. Scotto gave the presentation, "Aspects of Saturation and Ordering in Twelve-Tone Music," which consisted of two parts. One part of the talk dealt with aspects of saturation in twelve-tone structures. Specifically, Morris and Scotto sought to construct all-hexachord rows and/or rings, in a manner similar to Berg's all-interval row and Babbitt's all-trichord row. They presented various fifty-five note structures that contain all the hexachords, including fifty-five note rows which omit one pitch-class. The second part of the talk focused on self-replicating rows, which contain transformed versions of themselves embedded within the row. Arrays and cycles of twelve-tone operators play an important role in generating the latter type of structures.

[21] The special session attracted a certain amount of interest among the mathematicians at the meeting, and at times was attended by a number of outside interested parties. Based on its success, Peck and Baxter have proposed another special session for the American Mathematical Society/Mathematical Association of America national meeting, which will take place January 7-10, 2004, in Phoenix, Arizona. This subsequent session has been accepted by the conference program committee, and further information is available via the AMS website (http://www.ams.org), or by contacting Robert Peck (rpeck@lsu.edu).

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