

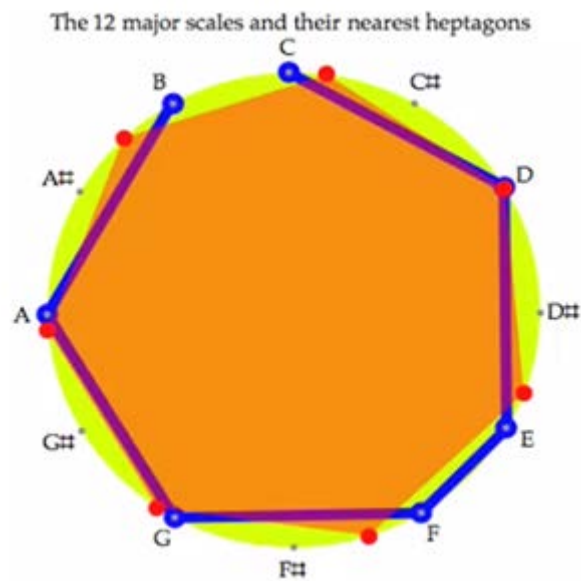
**MTO 15.2 Examples: Amiot, Discrete Fourier Transform and Bach's Good Temperament**

(Note: audio, video, and other interactive examples are only available online)  
<http://www.mtosmt.org/issues/mto.09.15.2/mto.09.15.2.amiot.php>

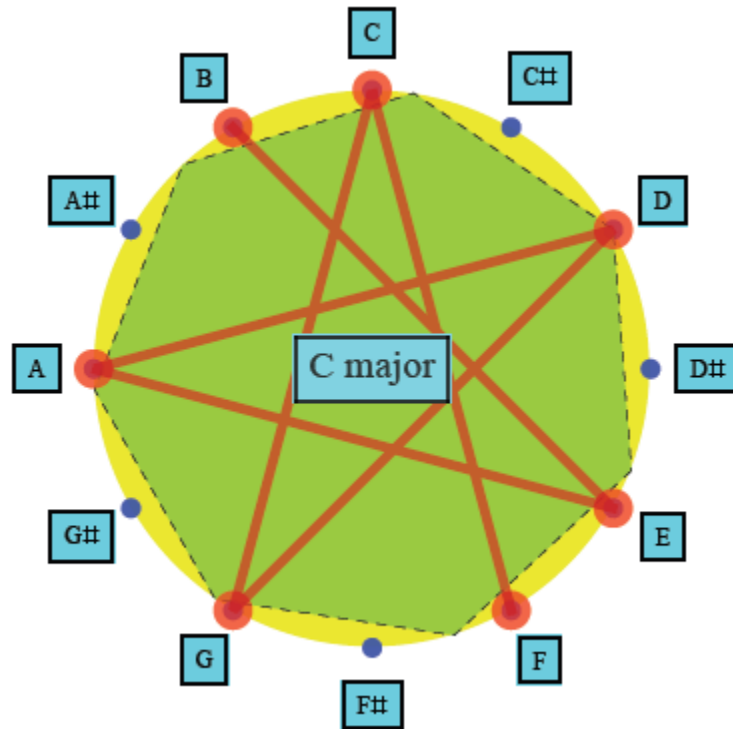
**Figure 1.** Bach's diagram on the first page of the WTC



**Video 1.**



**Figure 2.** Major scales are best approximations of regular heptagons



**Figure 3.** DFT of a major scale and a chromatic 7-note scale

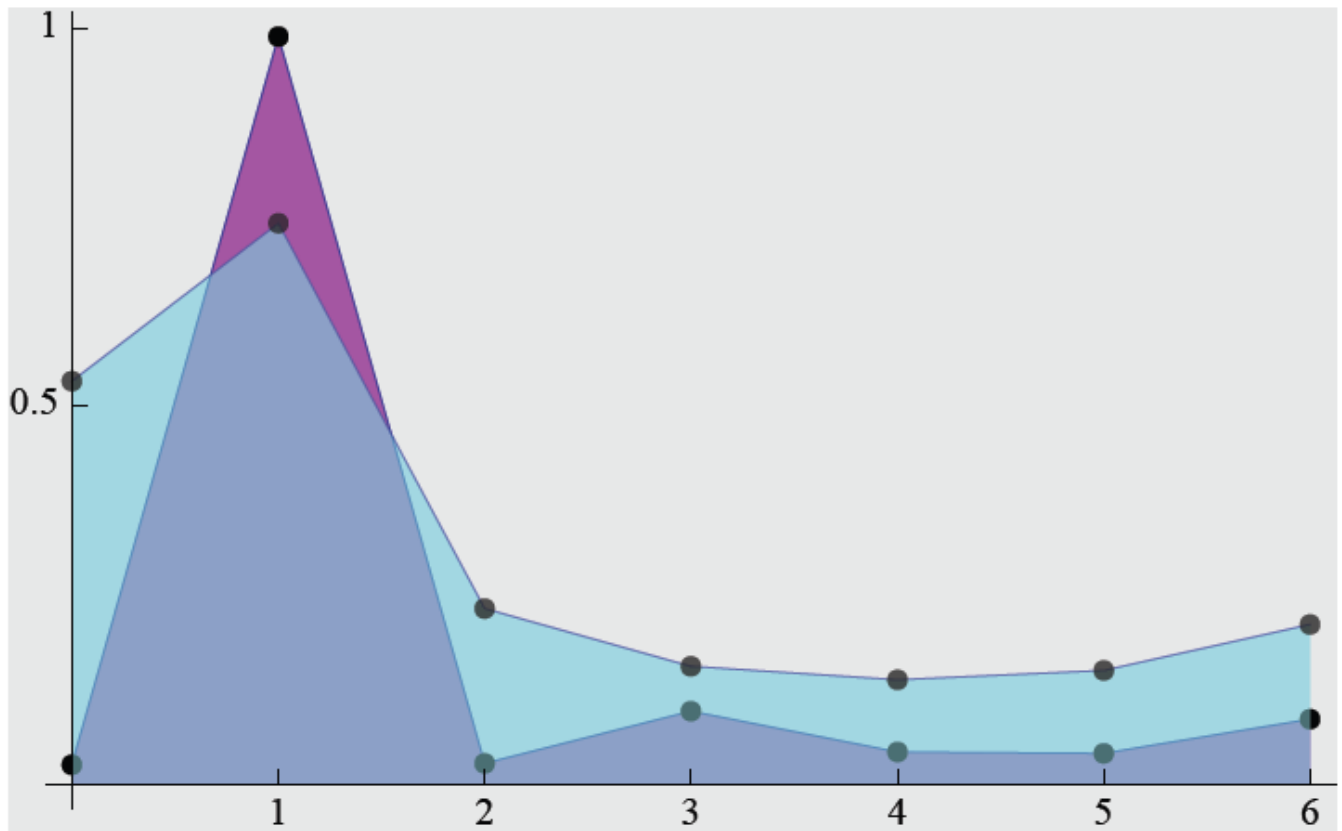


Figure 4. A program for computing MSS

```
fou1(gamme_) := Module[{n = Length[gamme]},
  N[ $\frac{e^{2i\pi\text{gamme}} \cdot \text{Table}[e^{-\frac{2i\pi k}{n}}, \{k, 0, n-1\}]}{n}$ ]]
consistency(scale_) := Module[{majScales, top, bottom},
  majScales = Table[|fou1(
    scale[[Sort[(r + {0, 2, 4, 5, 7, 9, 11} + 1) mod 12 /. 0 → 12]]] |,
    {r, 0, 11}];
  top = max(majScales); bottom = min(majScales);
  { $\frac{1}{\text{top} - \text{bottom}}$ , top, bottom}]
```

Table 1. Values of MSS for different tunings

	MSS	F	F#	G	G#	A	Bb	B	C	C#	D	Eb	E
Pythagore	141.572	0.	113.69	203.91	294.13	407.82	498.04	611.73	701.96	815.64	905.87	996.09	1109.8
Zarlino	58.9167	0.	111.73	203.91	315.64	386.31	498.04	590.22	701.96	813.69	884.36	1017.6	1088.3
Kirnberger2	146.869	0.	90.225	203.91	294.13	386.31	498.04	590.22	701.96	792.18	895.15	996.09	1088.3
Kirnberger3	164.332	0.	90.379	195.19	294.01	386.45	498.05	590.3	698.22	792.33	890.27	995.97	1088.2
Werkmeister1	180.602	0.	90.225	192.18	294.13	390.22	498.04	588.27	696.09	792.18	888.27	996.09	1092.2
Werkmeister2	119.663	0.	82.405	196.09	294.13	392.18	498.04	588.27	694.13	784.36	890.22	1003.9	1086.3
Werkmeister3	234.585	0.	96.09	203.91	300.	396.09	503.91	600.	701.96	792.18	900.	1002.	1098.
Werkmeister4	268.489	0.	90.661	196.2	298.07	395.17	498.04	594.92	697.54	792.62	893.21	1000.	1097.1
Werkmeister5	234.585	0	107.82	209.78	305.87	407.82	503.91	611.73	707.82	803.91	911.73	1007.8	1109.8
meanTone15	80.1584	0	113.69	194.53	308.21	389	502.74	616.4	697.26	810.95	891.79	1005.5	1086.3
meanTone16	117.405	0	109.78	196.09	305.87	392.18	501.96	611.73	698.04	807.2	894.13	1003.9	1090.2
Vallotti	164.255	0	94.13	196.09	298.04	392.18	501.96	592.18	698.04	796.09	894.13	1000	1090.2
BachLehman	260.483	0	103.91	200	305.87	403.91	501.96	603.91	698.04	807.82	901.96	1003.9	1103.9
Lindley	308.113	0.	106.	201.56	304.2	401.11	501.4	604.04	699.61	805.6	901.5	1002.8	1103.1
LindleyBis	361.731	0.	96.644	201.27	296.94	399.97	498.7	597.75	700.62	796.48	901.08	997.39	1098.9
Lindley94	224.272	0.	107.62	200.2	305.28	401.76	501.76	605.67	699.41	807.04	900.98	1003.5	1103.7
Sparschuh	292.595	0.	105.21	203.91	301.37	404.01	498.04	604.56	701.96	804.04	904.24	1000.2	1104.5
Lehman94	282.852	0	93.972	202.12	297.88	398.86	500.	595.6	700.16	795.93	900.49	999.84	1097.2

**Table 2.** MSS for O'Donnell-like tunings

	MSS	F	F#	G	G#	A	Bb	B	C	C#	D	Eb	E
BachLehman	260.483	0	103.91	200	305.87	403.91	501.96	603.91	698.04	807.82	901.96	1003.9	1103.9
O'Donnell	465.464	0	100.	201.96	300.	401.96	500.	598.04	700.	798.04	903.91	1000.	1100.
Neidhardt4	420.565	0.	101.96	200.	301.96	400.	498.04	603.91	698.04	803.91	900.	1000.	1100.
Neidhardt 1732 S.C.	258.472	0	96.09	201.96	296.09	401.96	500.	598.04	700.	796.09	903.91	998.04	1100.
Neidhardt 1732 B.C.	321.145	0	96.09	201.96	298.04	400.	498.04	596.09	701.96	798.04	901.96	998.04	1098.
Sorge	341.147	0	98.045	200.	296.09	400.	498.04	596.09	700.	796.09	901.96	996.09	1098.

**Figure 5.** The sum increases when the points are closer

