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Geometrical Methods in Recent Music Theory

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[1] The following remarks are prompted by Joti Rockwell's interesting article, "Birdcage Flights: A Perspective on Inter-Cardinality Voice Leading" (2009). My goal is not to take issue with Rockwell's specific claims but rather to underscore a few details that might escape the casual reader's attention. In particular, I want to stress three basic points.

1. Rockwell's discrete "birdcage graphs," which represent efficient voice leadings between chords of different sizes, can be embedded within the infinite-dimensional "OPC space" discussed by Callender, Quinn, and myself.[1] In general, discrete voice-leading graphs can always be embedded within the continuous spaces we describe.

2. Because of this, Rockwell's graphs inherit some of the complications of C space, and may distort voice-leading relationships among nonadjacent chords.

3. This is actually symptomatic of a more general issue affecting a variety of music-theoretical graphs. Roughly speaking, we have no guarantee that graphs whose *edges* refer to musical motions of a particular type will give rise to an intuitive or familiar notion of "distance" between nonadjacent chords.

[2] This last item is significant because theorists sometimes seem to endorse the following methodology. First, one selects some interesting domain of musical objects and some interesting set of motions among them. (For example, single-voice voice-leading between major and minor triads.) Second, one constructs a graph representing all of these motions between all the objects in question. Third, one interprets the resulting graph as providing a measure of distance. Thus, for example, one might use the graph to analyze music that moves between non-adjacent chords, or claim that larger leaps on the graph are musically disfavored in some way.

[3] This last step, however, involves a subtle leap. Consider, for example, the familiar *Tonnetz* (**Figure 1**).[2] Two chords are adjacent on this graph if they can be linked by what Cohn calls "parsimonious voice leading": voice leading in which a single voice moves, and it moves by just one or two semitones (Cohn 1996). However, larger distances in the space do not faithfully mirror voice-leading facts. On the *Tonnetz,* C major is two units away from F major but *three* units from F minor—even though it takes just two semitones of total motion to move from C major to F minor, and three to move from C major to F major (**Figure 2**). (This is precisely why F minor so often appears as a passing chord between F major and C major.) It follows that we cannot use the *Tonnetz* to explain the ubiquitous nineteenth-century IV-iv-I progression, in which the two-semitone motion $\hat{6}\rightarrow\hat{5}$ is broken into the semitonal steps $\hat{6}\rightarrow\flat\hat{6}\rightarrow\hat{5}$. More generally, it shows that *Tonnetz*-distances do not correspond to voice-leading distances in any straightforward way (Tymoczko 2009).

[4] Note that the problem persists even if we try to reinterpret the *Tonnetz* as representing *common tones* rather than voice leading: both F minor and E♭ minor are three *Tonnetz*-steps away from C major, even though C major and F minor have one common tone, while C major and E♭ minor have none. (As before, shorter distances are easier to interpret: two chords are

adjacent on the *Tonnetz* if they have two common tones, and any pair of chords that are *two* steps away will share exactly one common tone.) Thus, neither voice leading nor common tones allow us to characterize *Tonnetz* distances precisely. We seem forced to say that *Tonnetz*-distances represent simply *the number of parsimonious moves* needed to get from one chord to another—and not some more familiar music-theoretical quality.

[5] From this point of view, there is a fundamental difference between the *Tonnetz* and Douthett and Steinbach's "Cube Dance" (Douthett and Steinbach 1998). (**Figure 3**) Like the *Tonnetz*, "Cube Dance" depicts a collection of local moves, in this case the single-semitone voice leadings between major, minor, and augmented triads. (On Douthett and Steinbach's graph, descending semitonal motion is represented by clockwise steps, while ascending semitonal motion is represented by counterclockwise steps.) Unlike the *Tonnetz*, however, "Cube Dance" *also* faithfully models voice-leading distances between non-adjacent chords; in fact, *any* clockwise or counterclockwise path on "Cube Dance" can be associated with a particular voice leading, with the length of the path (as measured in edges) corresponding to the size of the voice leading (as measured according to "taxicab distance," or the total number of semitonal steps in all voices). Compared to the *Tonnetz*, then, "Cube Dance" is a more genuinely geometrical, modeling musically familiar distances between nonadjacent objects. To be sure, this distinction may not be intuitively obvious on first inspection. In fact, the difference between these two sorts of graphs only became clear after theorists discovered how to construct the *n*-dimensional spaces representing all possible *n*-voice voice-leadings between all possible *n*-note chords.[(3)]

[6] With this distinction in mind, let's now turn to Rockwell's "birdcage graphs." **Figure 4**, which is reproduced from Rockwell's article, connects dominant seventh chords and minor triads if they can be linked by voice leading in which two voices move by semitone. Thus A♭^{7} and C minor are adjacent because they can be linked the "augmented sixth" voice leading (A♭, C, E♭, G♭)→(G, C, E♭, G), and C minor and A♭ minor are adjacent because they can be connected by the voice leading (C, E♭, G)→(C♭, E♭, A♭). But notice that the larger distances are not so easy to interpret: A♭^{7} and C^7 are both equidistant from A♭ minor, even though the minimal voice leading from A♭^{7} to A♭ minor, (A♭, C, E♭, G♭)→(A♭, C♭, E♭, A♭), involves three semitones of total motion, while the minimal voice leading from C^7 to A♭ minor, (C, E, G, B♭)→(C♭, E♭, A♭, C♭), involves four. Thus, though single-edge motions on Rockwell's graph refer to a particular sort of voice leading (single-semitone motion in two distinct voices), the two-edge motions do not. **Figure 5** shows that this is because there are various ways to combine the graph's voice leadings: in Figure 5(a) the two motions in the bass cancel out, while in Figure 5(b) no such cancellation occurs.

[7] The problem here is symptomatic of a larger issue, namely that it is difficult to represent voice-leading relations between chords of different sizes. Importantly, *this is as true for discrete graphs as it is for "C space" in all its infinite-dimensional glory*. For another example, consider **Figure 6**, which faithfully depicts single-semitone voice leadings between chromatic clusters, semitones, and single notes. (On this graph, two chords are adjacent if they can be linked by a voice-leading in which some notes are doubled, and in which one voice moves by one semitone.) However, the larger distances again diverge from voice leading distances: the graph depicts {F♯, G, A♭} (=678) and {C♯, D, E♭} (=123) as being seven edges away from one another, even though the minimal voice leading between them involves at least fifteen total semitones of motion.[(4)] A central conclusion of Callender, Quinn, and Tymoczko (2008) is that similar problems will *inevitably* appear as we try to subsume more and more chords (of differing sizes) within our graphs: to obtain completeness without sacrificing contrapuntal fidelity, we must restrict ourselves to multisets of some particular size.

[8] The broader moral is that we should take care to distinguish two different sorts of music-theoretical models. The first represents only a collection of local relationships: at any point in the space, it shows us all the available "moves" of a certain kind. (Here we might think of a subway map that shows which stations are adjacent to any other station.) The second type may do this as well, but it also captures some familiar notion of distance between *all* the objects it represents—even those that are not immediately adjacent on the graph. The important point is that we have no guarantee that any particular notion of musical distance will necessarily give rise to any coherent geometry of this second sort. (It is, for example, quite difficult to construct a geometrical space whose points represent major and minor triads, and in which distance represents the number of common tones.) Nor, conversely, can we be sure that a particular collection of local moves give rise to a familiar notion of musical distance. From this point of view, the remarkable fact is that we *can* construct coherent geometries in the special case where we are concerned with voice leading among multisets of a fixed size.

[9] Of course, none of this is meant as a criticism of Rockwell, or as an objection to his useful graphical constructions. My point, rather, is that we need caution when interpreting the sorts of structures he describes. Some graphs are useful primarily insofar as they depict a collection of local moves, while others give rise to a more complex geometry, and the difference may not always apparent upon casual inspection.

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Footnotes

1. OPC space is the geometrical space that is formed when we discard the octave, order, and multiplicity of groups of notes. From this point of view (C4, E4, G4) is equivalent to (E3, G4, G5, C2, E4). The space is infinite dimensional because it contains sequences of arbitrary length. See Callender, Quinn, and Tymoczko 2008.
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2. See Mooney 1996, Hyer 2002, and Cohn 1996 for the discussion of the *Tonnetz* and its history.
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3. See Tymoczko 2006, 2009 and 2010, the last of which shows that both Douthett and Steinbach's "Power Towers" and my own "scale lattice" (Tymoczko 2004) faithfully reflect voice-leading distances between non-adjacent chords.
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4. The issue here is that the shortest graphical path between {F♯, G, A♭} and {C♯, D, E♭} involves a series of voice leadings with different numbers of voices: one can collapse the three-note {F♯, G, A♭} onto {F♯, G} by a single edge, representing the three-voice voice leading (F♯, G, A♭)→(F♯, G, G); {F♯, G} can then be collapsed to F♯ by an edge that represents the *two-voice* voice leading (F♯, G)→(F♯, F♯). Moving F♯ to E♭ then takes three more edges, representing the *one-voice* voice-leading F♯→E♭. (This E♭ then expands to {C♯, D, E♭} by two more edges representing two- and three-voice voice leadings.) Since these component voice leadings have different numbers of voices, they do not combine to form a three-voice voice leading in which the voices move by a collective total of seven semitones. This is just a discrete version of the example discussed in Callender, Quinn, and Tymoczko (2008), supplementary section 4.
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