

MTO 20.1 Examples: Callender, Performing the Irrational

(Note: audio, video, and other interactive examples are only available online)

<http://www.mtosmt.org/issues/mto.14.20.1/mto.14.20.1.callender.php>

Table 1. Arrangements of Nancarrow's primarily canonic studies

Study	Ratios	Arrangement(s)
No. 14	4:5	Helena Bugallo, Raaf Hekkema, Ivar Mikhashoff, Dorrance Stalvey
No. 15	3:4	Hekkema, Mikhashoff (multiple)
No. 16	3:5	Hekkema, Erik Oña, James Tenney
No. 17	12:15:20	Mikhashoff
No. 18	3:4	Hekkema, Mikhashoff, Oña
No. 19	12:15:20	Bugallo, Mikhashoff
No. 21	"X"	Dominic Murcott (with two conductors using click tracks)
No. 22	$1\%:1\frac{1}{2}\%:2\frac{1}{4}\%$	none
No. 24	14:15:16	none
No. 26	1:1	Bugallo, Wolfgang Heisig, Hekkema, Nancarrow, Matt Rogers, Stalvey
No. 27	5% : 6% : 8% : 11%	none
No. 31	21:24:25	Paul Usher
No. 32	5:6:7:8	Bugallo, Stalvey
No. 33	$2:\sqrt{2}$	Usher
No. 34	9:10:11	Nancarrow
No. 36	17:18:19:20	none
No. 37	$1:\frac{15}{14}:\frac{9}{8}:\frac{6}{5}:\frac{5}{4}:\frac{4}{3}:\frac{7}{5}:\frac{3}{2}:\frac{8}{5}:\frac{5}{3}:\frac{7}{4}:\frac{15}{8}$	none
No. 40	$e:\pi$	none
No. 41a	$\frac{1}{\sqrt{\pi}}:\sqrt{\frac{2}{3}}$	none
No. 41b	$\frac{1}{\sqrt[3]{\pi}}:\sqrt[3]{\frac{13}{16}}$	none
No. 43	24:25	none
No. 44	Indeterminate tempo ratio	Bugallo
No. 47	5:7	none
No. 48	60:61	none
No. 49	4:5:6	Thomas Adès

Figure 1. 12:15:20 tempo ratio as a polyrhythm in a single notated tempo



Figure 2. Opening of Nancarrow's Study No. 33

$\text{♩} = 280$

$\text{♩} = 140 \times \sqrt{2}$

Figure 3. Nearly aligned chords in the opening of Nancarrow's Study 33

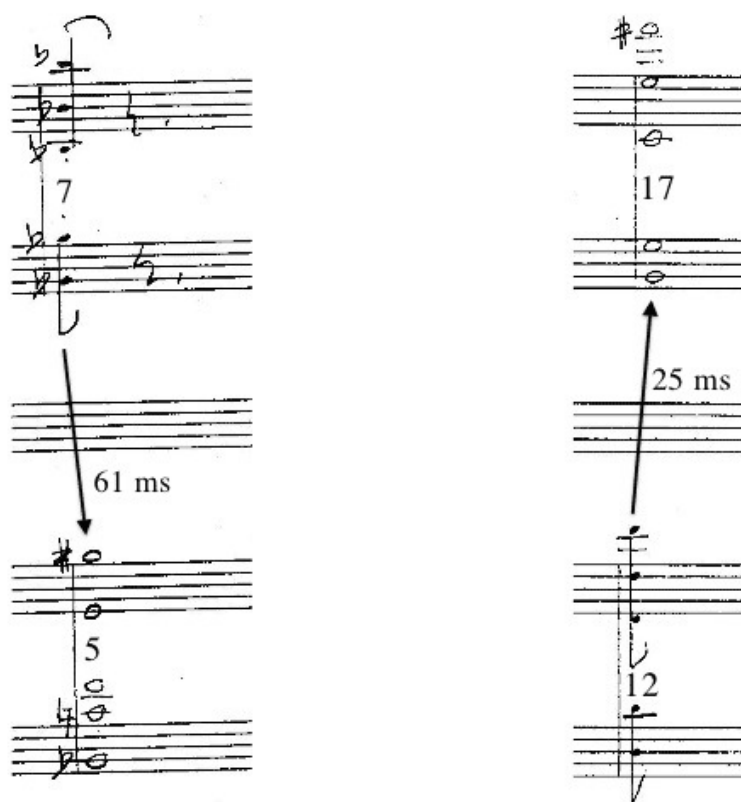


Figure 4. Opening of Usher's arrangement of Study 33

$\text{♩} = \text{ca. } 64$

I.

Violin I

viola

Violin II

violoncello

f *ff* *f* *ff* *f* *ff* *f* *ff*

gl. *gl.* *gl.* *gl.* *gl.* *gl.* *gl.* *gl.*

0 1 2 3 0 3 0 3

$\frac{17}{12}$ $\approx \sqrt{2}$ $2 \times \frac{17}{12}$ $\approx 2\sqrt{2}$

Figure 5. Nearly aligned chords in Usher's arrangement

The figure displays two systems of musical notation, each consisting of four staves. The left system features the following elements:

- Staff 1: A single eighth note with a flat (B $\flat$).
- Staff 2: A whole note with a flat (B $\flat$), with the number 7 written below it.
- Staff 3: A whole note with a flat (B $\flat$), with the expression $7 + \frac{1}{12}$ written below it.
- Staff 4: A triplet of eighth notes (F $\sharp$, A, B $\flat$), with the expression $\approx 5\sqrt{2}$ written below it.

The right system features the following elements:

- Staff 1: A complex chord structure with multiple notes and accidentals, including a sharp (F $\sharp$).
- Staff 2: A whole note with a flat (B $\flat$), with the number 17 written below it.
- Staff 3: A triplet of eighth notes (F $\sharp$, A, B $\flat$), with the expression $\approx 12\sqrt{2}$ written below it.
- Staff 4: A triplet of eighth notes (F $\sharp$, A, B $\flat$).

Figure 6. Usher's arrangement of Study No. 33, Canon 3

(Canon 3) 5

The musical score is arranged in systems, each containing staves for Violin I (Vln I), Violin II (Vln II), and Viola/Celli (Vc.). The key signature is one flat (B-flat major or D minor), and the time signature is 2/4. The score begins at measure 51, marked with a double bar line and a repeat sign. Measures 51-55 are marked with a piano (*p*) dynamic. The score includes various musical notations such as slurs, ties, and articulation marks. Measure numbers 51, 56, 59, 63, 67, and 71 are indicated at the start of their respective systems. The score concludes at measure 71.

Figure 7. Alternate notation for violin 2 in Study No. 33, Canon 3, measure 55

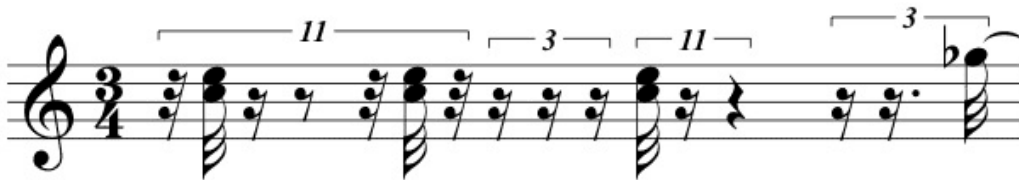


Figure 8. 7:5 tempo ratio with a convergence period of $\frac{7}{8}$ seconds

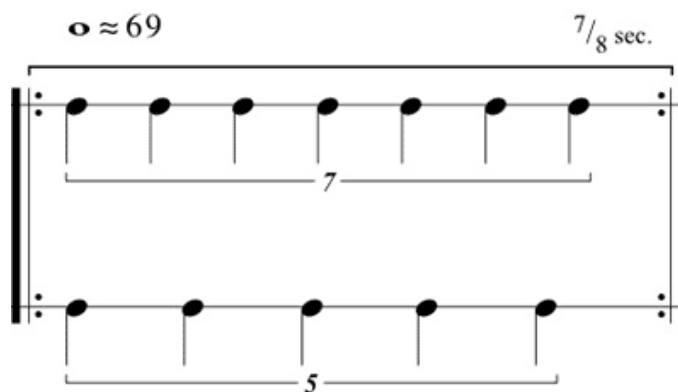
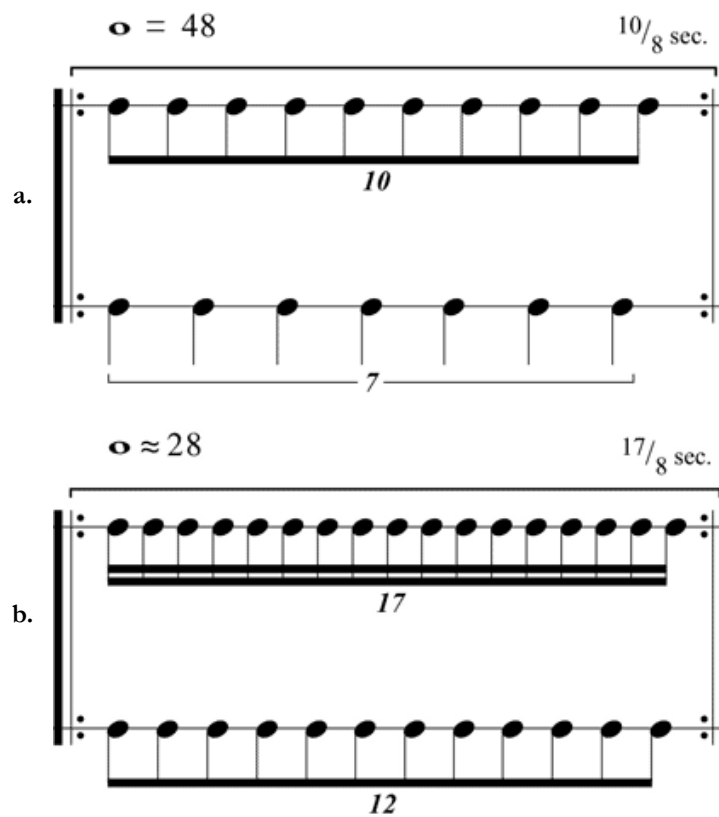


Figure 9. Convergence periods for (a) 10:7 tempo ratio, (b) 17:12 tempo ratio, and (c) 24:17 tempo ratio



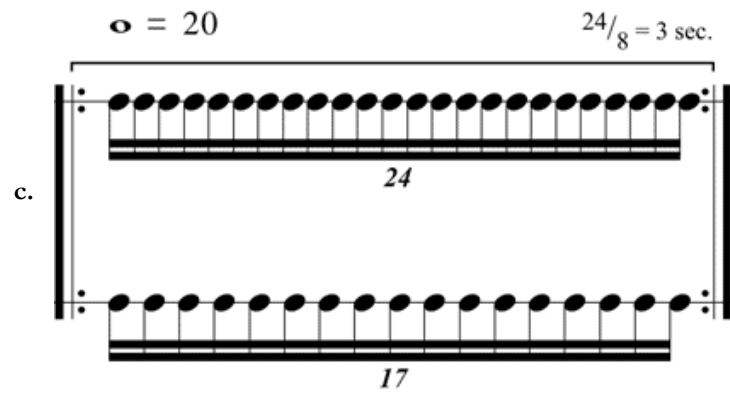


Figure 10. 5+6+6 division in the tempo ratio $\frac{24}{17}$

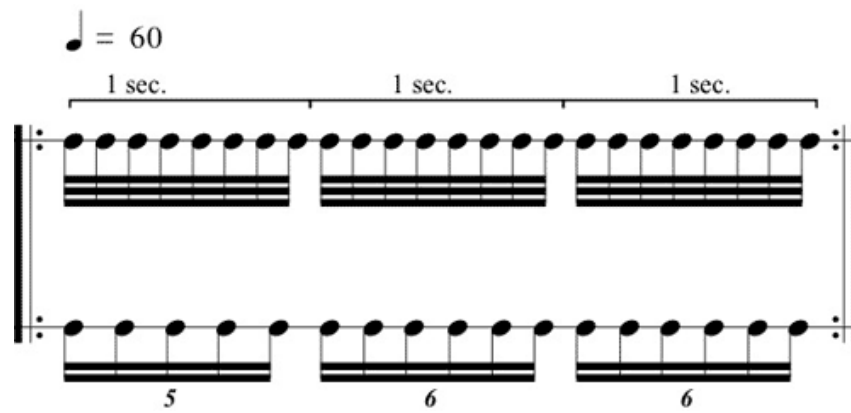


Figure 11. 11+6 division in the tempo ratio $\frac{24}{17}$

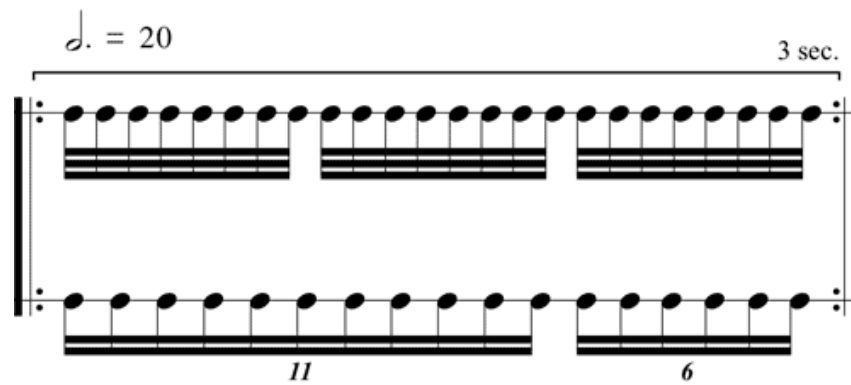


Figure 12. Hypothetical version of canon 3, approximating tempo ratio

Figure 13. Hypothetical version of canon 3, approximating beat placement

Figure 14. Nancarrow's approximation of $\sqrt{2}$, Study No. 33, systems 56–60

The first few durations are marked as multiples of a sixteenth note

Handwritten musical score for Nancarrow's Study No. 33, systems 56–60. The score is written on ten staves, with the first two staves of each system being a grand staff (treble and bass clef). The notation includes various rhythmic values, accidentals, and dynamic markings like "PP" and "pp". Numbers 8, 16, 4, 8, 12, 11, 23, 5, 13, 5, 11, 17 are placed above or below notes, indicating durations as multiples of a sixteenth note. The tempo is marked as quarter note = 280 ($\text{♩} = 280$). The key signature has one sharp (F#).

Figure 15. (a) Converging-diverging canon, (b) Diverging-converging canon, and (c) Abstract canonic structure of Study

33

These three structural diagrams are adapted from Gann 1995

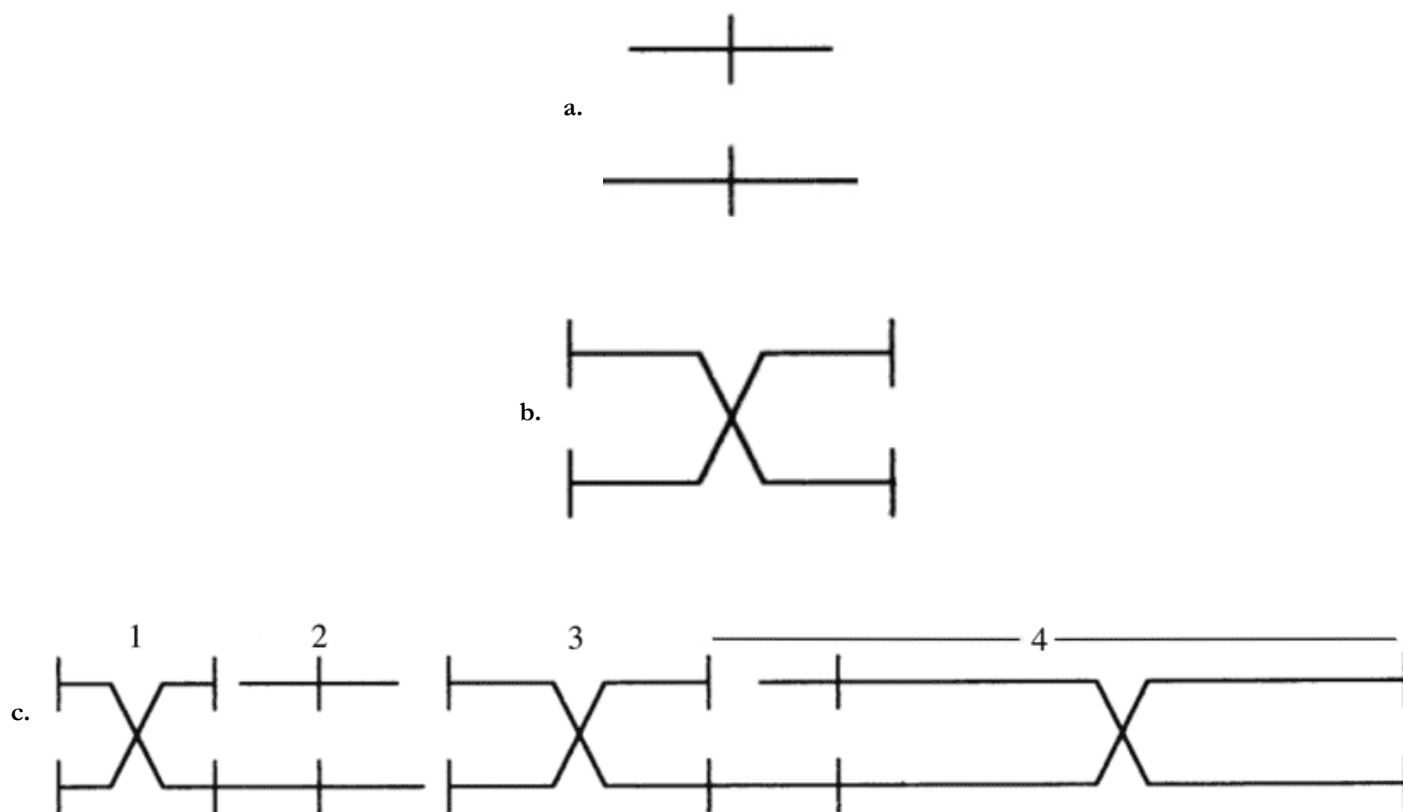


Figure 16. Tempo exchange, canon 1, Nancarrow

$J = 140 \times \sqrt{2}$

25 26 27

exchange gap $\approx 1/8$ whole notes

$J = 280$

$18\sqrt{2}$ $19\sqrt{2}$

Figure 17. Realization of the tempo exchange in canon 1 that yields an unnecessarily complex rhythmic notation after the exchange

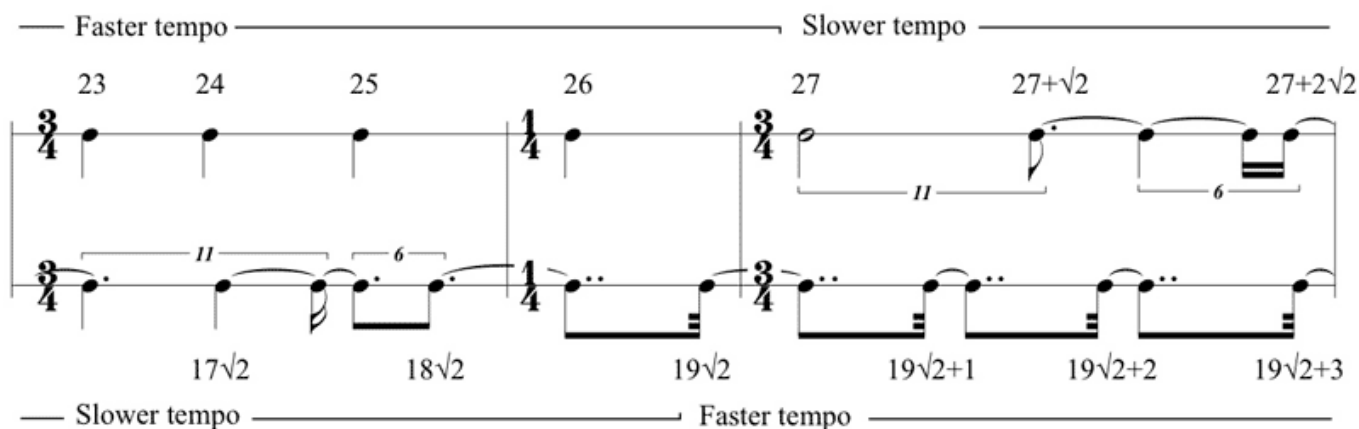


Figure 18. A different realization of the same tempo exchange that yields a much simpler rhythmic notation

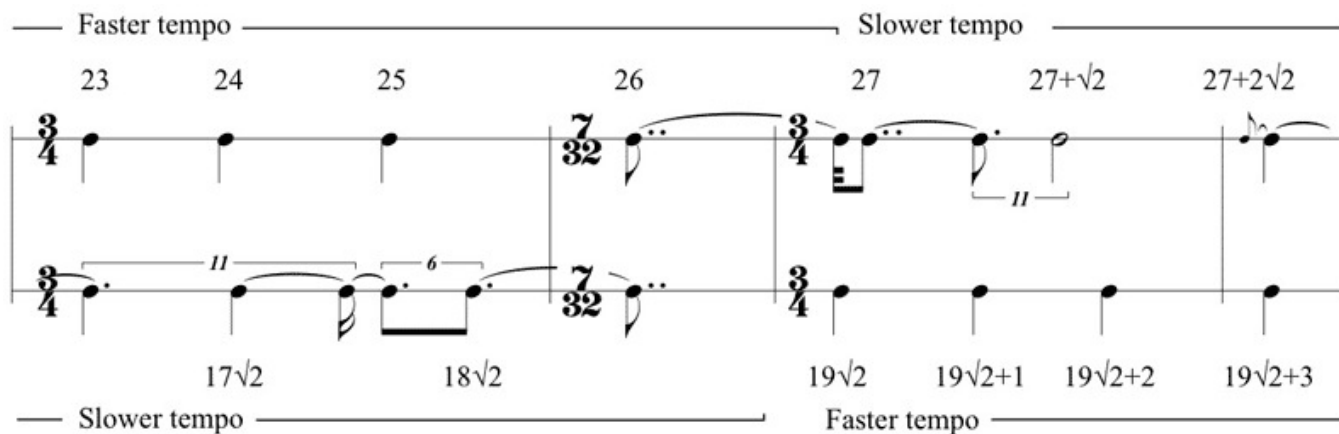


Figure 19. Tempo exchange, canon 1, Usher

Faster tempo

Slower tempo

beats:

15 $\sqrt{2}$

16 $\sqrt{2}$

17 $\sqrt{2}$

18 $\sqrt{2}$

19 $\sqrt{2}$

19 $\sqrt{2}$ + 1

exchange gap ≈ 18 ms

Figure 20. Tempo exchange, canon 3, Nancarrow

Faster tempo

Slower tempo

exchange gap ≈ 18 ms

Figure 21. Tempo exchange, canon 3, Usher

The musical score illustrates a tempo exchange in Canon 3 by Usher. It features three staves: Violin I (vln I), Viola (vlna), and Violin II (vln II). The score is divided into two sections by a vertical line, with tempo changes indicated above and below the staves.

Violin I (vln I): The first section, labeled "Faster tempo", starts at measure 75 in 2/4 time. The second section, labeled "Slower tempo", begins with a 3-measure rest followed by an 11:8 ratio, then continues in 3/8 time. An annotation "147 sixteenths" points to the first measure of the slower tempo section.

Viola (vlna): The first section is labeled "Slower tempo" and the second section is labeled "Faster tempo". The Viola part consists of rests in both sections, with the time signature changing from 2/4 to 3/8 at the tempo exchange.

Violin II (vln II): The first section, labeled "Slower tempo", begins with an 11:8 ratio followed by a 3-measure rest. The second section, labeled "Faster tempo", begins with a 104√2 sixteenths annotation pointing to the first measure, then continues in 3/8 time.

Figure 22. Tempo exchange, canon 4, Nancarrow

$\text{♩} = 140 \times \sqrt{2}$

$\text{♩} = 280$

exchange gap
 $\approx 6/5$ eighth notes

$(\frac{2}{4})$ 898.5 eighths

$(\frac{2}{4})$ 634.5 $\sqrt{2}$ eighths

Figure 23. Tempo exchange, canon 4, Usher

The musical score is divided into two systems. The first system (measures 172-174) features Vln I, Vla, and Vc. staves. Vln I and Vc. have 11:8 and 3 ratios indicated above them. The second system (measures 175-177) features Vln I, Vla, Vln II, and Vc. staves. Vln I and Vln II have 40 and 2/4 time signatures indicated. Vln I has a tempo value of ≈ 897.3 thirty-seconds. Vla has a tempo value of 896.5 thirty-seconds. Vln II has a tempo value of $\approx 634.5\sqrt{2}$ thirty-seconds. Vc. has a tempo value of $\approx 634.5\sqrt{2}$ thirty-seconds. The score includes various musical notations such as notes, rests, and dynamic markings like *f*.

Figure 24. Opening of canon 2, Nancarrow

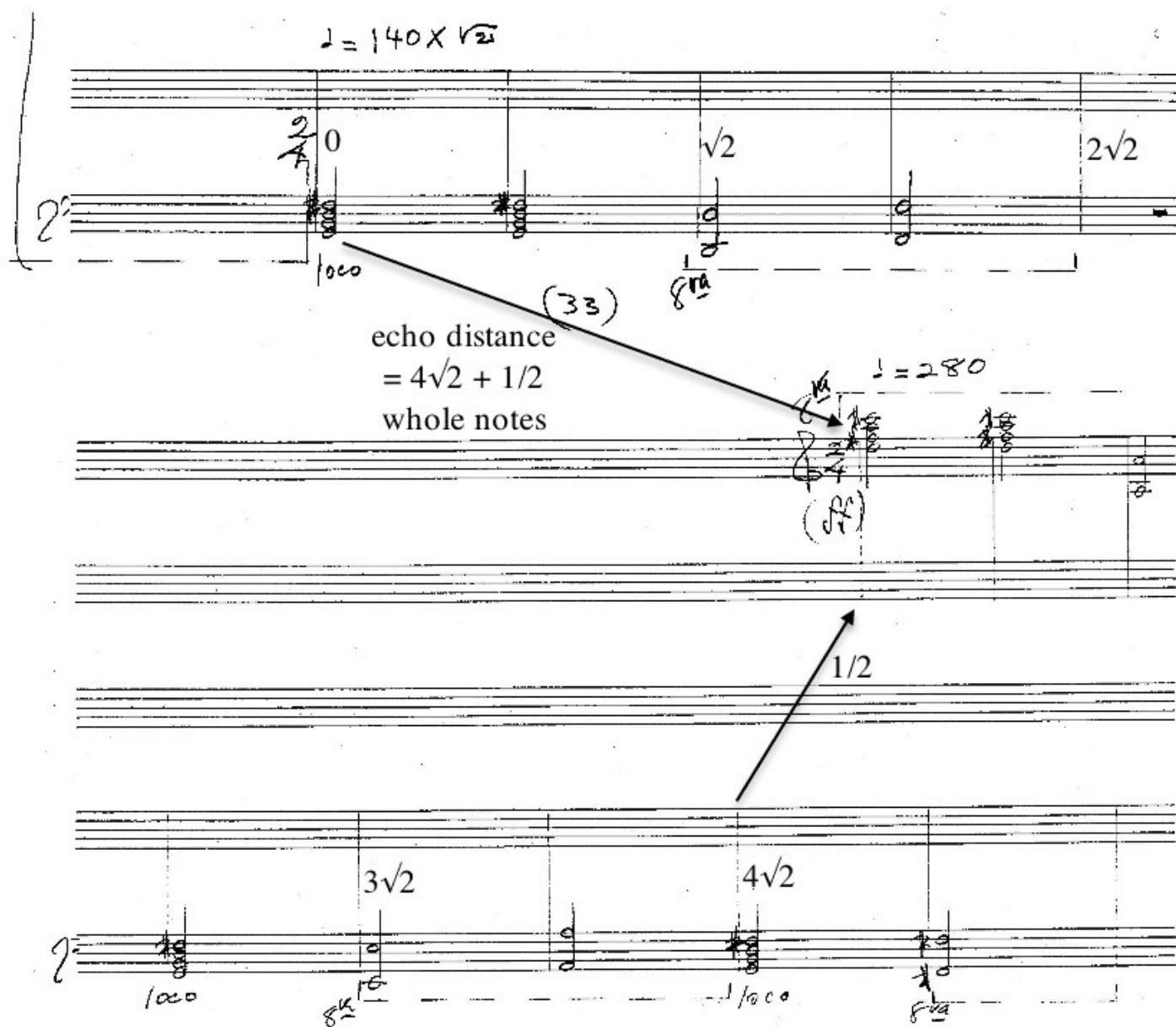


Figure 25. Opening of canon 2, Usher

The musical score is written for four instruments: Violin I (vln I), Viola (vlna), Violin II (vln II), and Violoncello (vc.). The time signature is 2/4. The key signature has one sharp (F#).

Violin I (vln I): The staff begins with a whole rest in the first measure (marked 3/20) and another whole rest in the second measure (marked 1/8). At measure 31, it enters with a half note F#4, marked *f*. Above the staff, the Roman numerals II, III, and IV are indicated.

Viola (vlna): The staff begins with a whole rest in the first measure (marked 3/20) and another whole rest in the second measure (marked 1/8). At measure 31, it enters with a half note F#4, marked *mf*. Above the staff, the Roman numerals I and II are indicated.

Violin II (vln II): The staff begins with a half note F#4 in the first measure (marked 3/20, *ff*) and a half note F#4 in the second measure (marked 1/8, *ff*). A bracket labeled "5" spans the two measures. At measure 31, it enters with a half note F#4, marked *ff*. Above the staff, the Roman numerals 11:8 and 11:8 are indicated.

Violoncello (vc.): The staff begins with a half note F#4 in the first measure (marked 3/20, *ff*) and a half note F#4 in the second measure (marked 1/8, *ff*). A bracket labeled "3" spans the two measures. At measure 31, it enters with a half note F#4, marked *ff*. Above the staff, the Roman numerals 3 and 11 are indicated.

Annotations: A diagonal arrow points from the first measure of Violin II to the first measure of Violin I. Text below the arrow reads: "echo distance approximately 6 and 1/10 quarters".

Figure 26. Opening of canon 4, Nancarrow

Handwritten musical score for the opening of Canon 4 by John Cage, featuring complex rhythmic notation and mathematical calculations.

Top Staff: A short musical phrase with notes labeled $79\sqrt{2}$, $78\sqrt{2}$, and $77\sqrt{2}$. Above the staff is the handwritten formula $1 = 140 \times \sqrt{2}$.

Middle Staff: A longer musical phrase starting with a note labeled $76\sqrt{2}$. It includes various rhythmic markings such as $\text{p} \text{ra}$ and loco .

Text: Below the middle staff, the following calculation is written:
 echo distance = $79\sqrt{2} - 79$
 $\approx 32 \frac{2}{3}$ quarter notes or $8 \frac{1}{6}$ whole notes

Bottom Staff: A musical phrase with notes labeled 79, 78, 77, and 76. Above these notes are the numbers 8, 7, 6, and 5 respectively. A vertical line connects the $79\sqrt{2}$ note in the middle staff to the 79 note in the bottom staff, with the label $1 = 280$ next to it. The bottom staff also includes markings like loco and $8\sqrt{2}$.

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Figure 27. Opening of canon 4, Usher

The musical score for the opening of Canon 4 by Usher is presented in four staves. The top staff, labeled 'arco', begins in 1/24 time and transitions to 2/4 time, marked with a forte (*fff*) dynamic. It features a series of rhythmic patterns with time signatures 12:8, 11:8, 11:8, and 11: over the measures. The second staff, labeled 'vln I', starts at measure 101 in 12:8 time, marked *ff*, and includes a 'b' (flat) at the end. The third staff, labeled 'vln I', starts at measure 103 in 5/8 time, marked *ff*, and includes a '3' (triple) at the end. The fourth staff, labeled 'vla', starts at measure 103 in 12:8 time, marked *ff*, and includes a '3' (triple) at the end. A diagonal arrow points from the first staff to the third, with the text 'echo distance = 8 1/6 quarter notes' indicating the interval between the two parts.

Figure 28. Opening of canon 3 with Arditti String Quartet recording

(Canon 3)

5

The musical score for Canon 3, measures 51-74, is presented in three systems. Each system contains staves for Violin I (vln I), Violin II (vln II), and Viola (vc.). The time signature is 2/4. The key signature has one flat (B-flat). The score includes various musical notations such as notes, rests, and dynamic markings. The first system (measures 51-55) shows the beginning of the canon with a forte (f) dynamic. The second system (measures 56-58) continues the canon. The third system (measures 59-62) shows the continuation of the canon. The fourth system (measures 63-66) includes measures 63-66. The fifth system (measures 67-70) includes measures 67-70. The sixth system (measures 71-74) includes measures 71-74. The score includes various musical notations such as notes, rests, and dynamic markings. The first system (measures 51-55) shows the beginning of the canon with a forte (f) dynamic. The second system (measures 56-58) continues the canon. The third system (measures 59-62) shows the continuation of the canon. The fourth system (measures 63-66) includes measures 63-66. The fifth system (measures 67-70) includes measures 67-70. The sixth system (measures 71-74) includes measures 71-74. The score includes various musical notations such as notes, rests, and dynamic markings.

Figure 29. Graph of beats with respect to time and the resulting trend lines in the recording of the opening of canon 3

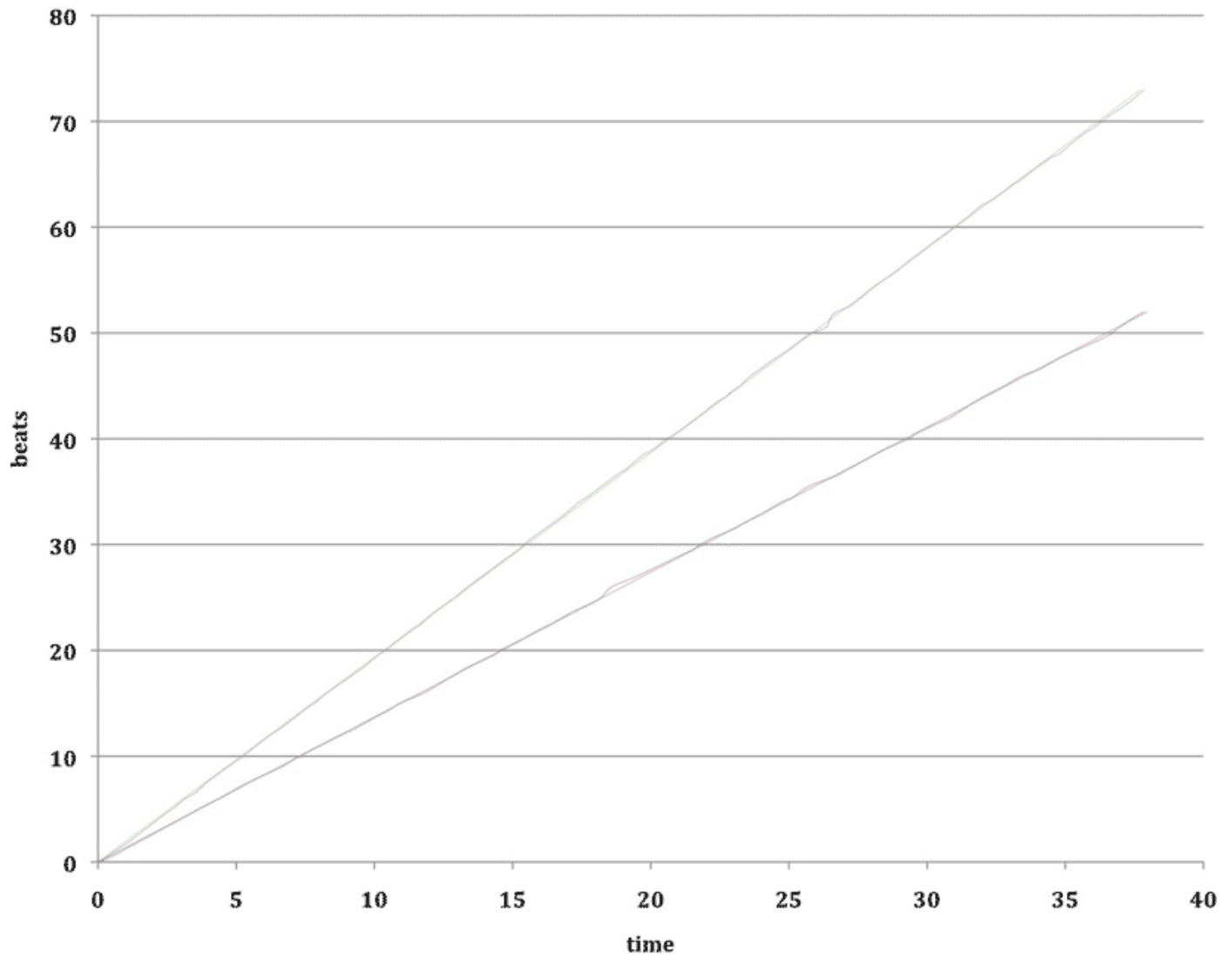


Figure 30. Inter-onset intervals in the original and recorded versions of the opening of canon 3

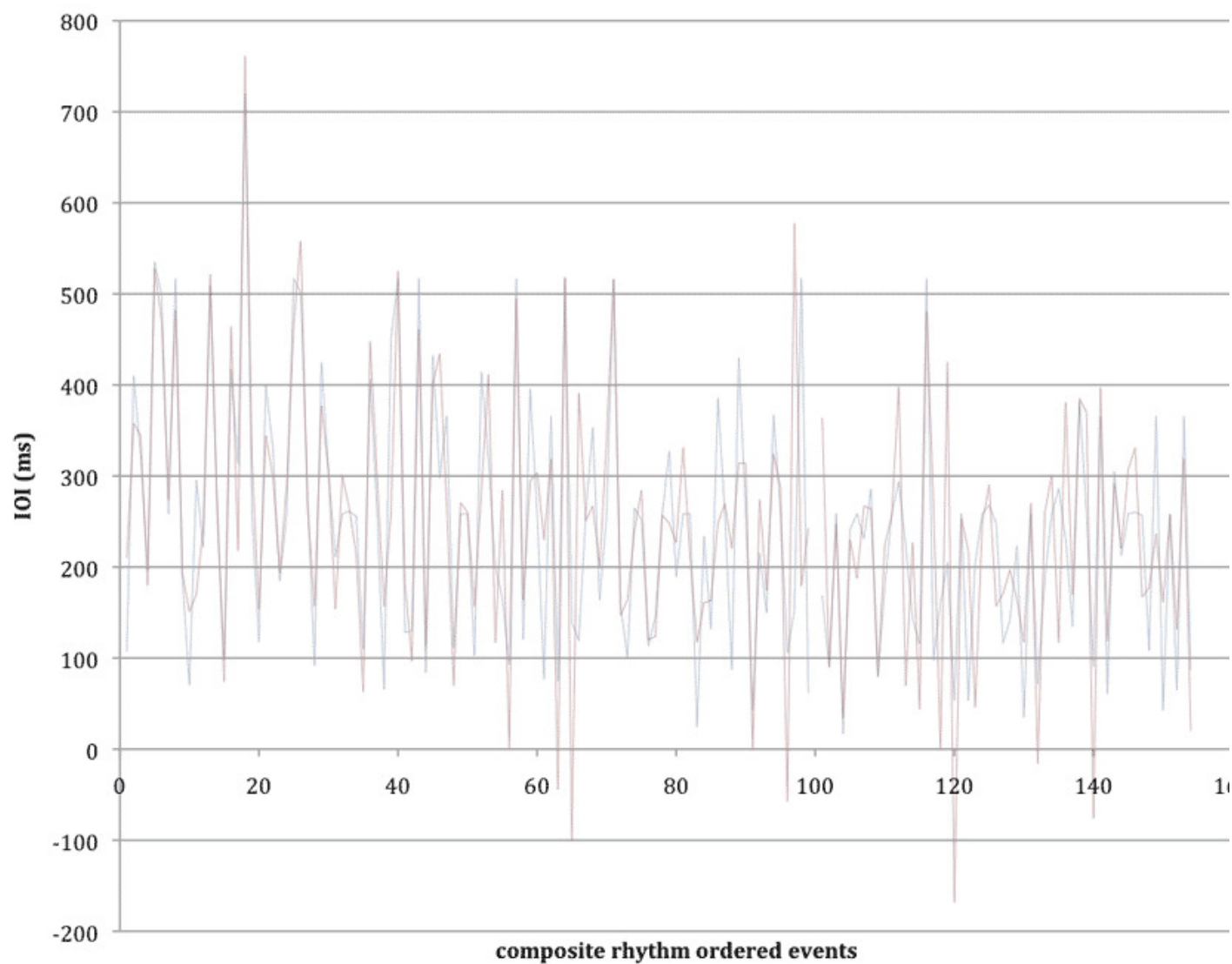


Figure 31. (a) First violin measure 68 as performed and (b) as notated

