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MTO 4.4 Examples: Scholtz, Algorithms for Tunings and Temperaments

(Note: audio, video, and other interactive examples are only available online)
http://www.mtosmt.org/issues/mto.98.4.4/mto.98.4.4.scholtz.php

Figure 1. A Section of the Chain of Fifths

Bbb-Fb-Cb-Gb-Db-Ab-Eb-Bb-F-C-G-D-A-E-B-F#-C#-G#-D#-A#-E#-B#

Figure 2. Six Diatonic Scales in a Chromatic Chain From E♭ to G♯

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Bb major, G-minor        Eb-Bb-F-C-G-D-A
F  major, D-minor           Bb-F-C-G-D-A-E
C  major, A-minor              F-C-G-D-A-E-B
G  major, E-minor                 C-G-D-A-E-B-F#
D  major, B-minor                    G-D-A-E-B-F#-C#
A  major, F#-minor                      D-A-E-B-F#-C#-G#
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Figure 3. Harmonic Intervals Ordered From the Chain of Fifths

Ascending Harmonic Intervals

Links	Notes	Left to Right	Right to Left
1	C–G	Perfect fifth	Perfect fourth
2	C–D	Major second	Minor seventh
3	C–A	Major sixth	Minor third
4	C–E	Major third	Minor sixth
5	C–B	Major seventh or minor second	Diatonic semitone
6	C–F#	Augmented Fourth or tritone	Diminished Fifth
7	C–C#	Chromatic semitone	Diminished octave
8	C–G#	Augmented fifth	Diminished fourth
9	C–D#	Augmented second	Diminished minor seventh
10	C–A#	Augmented sixth	Diminished minor third
11	C–E#	Augmented third	Diminished minor sixth

Figure 4. Pythagorean Tuning

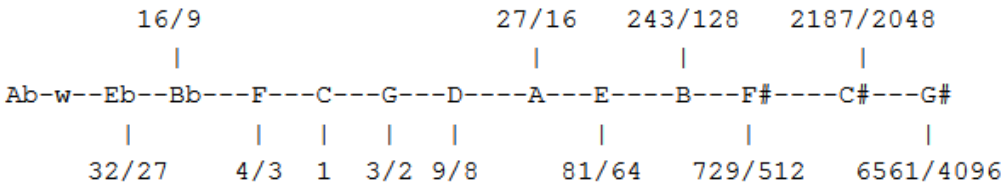

Figure 5. Schematic of Equal Temperament

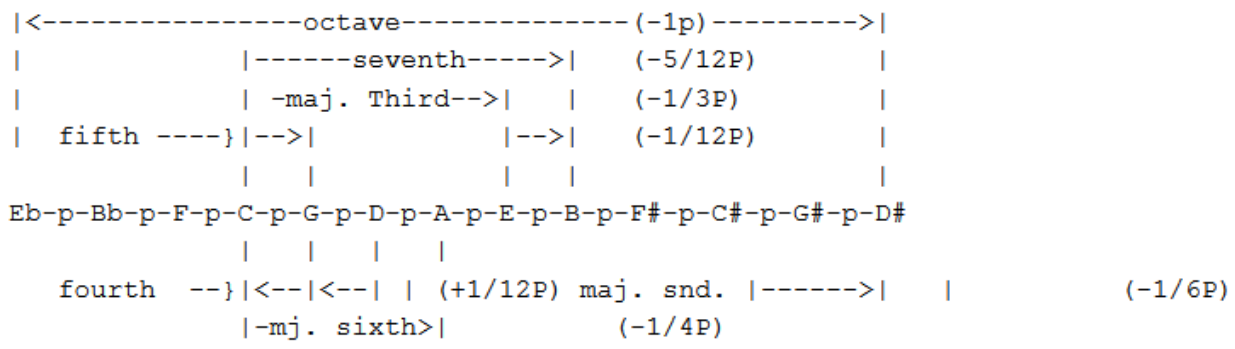

Figure 6. Enharmonic Pairs in Equal Temperament

C	G	D	A	E	B	F#	C#	G#	D#	A#	E#	B#
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C

Figure 7. Equal Temperament

Note	Tuning	Cents	
G#/Ab	1.587401	800	
C#/Db	1.059463	100	(1300–1200)
F#/Gb	1.414214	600	(1800–1200)
B	1.887749	1100	
E	1.259921	400	(1600–1200)
A	1.6817793	900	
D	1.122462	200	(1400–1200)
G	1.498307	700	
C	1.0	0/1200	
F	1.334840	500	(1700–1200)
Bb/A#	1.781797	1000	
Eb/D#	1.189207	300	

Figure 8. Dissonant Internal Intervals in Just Intonation Scale

F	C	G	D	k	A	E	B
4/3	1	3/2	9/8		5/3	5/4	15/8

D–A = 5/3 x 8/9 = 40/27, A–D = 27/20
D–F = 4/3 x 8/9 = 32/27, F–D = 27/16

Figure 9. Just scales in B♭, F, C and G

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  Eb----Bb----F----C--k--G-----D---A
32/27  16/9  4/3   1   27/20 10/9 5/3

        Bb----F----C----G--k--D----A----E
       16/9  4/3   1   3/2  10/9  5/3  5/4

             F----C----G----D--k--A----E----B
            4/3   1   3/2  9/8   5/3  5/4 15/8

                  C----G----D----A--k--E----B----F#
                  1   3/2  9/8 27/16  5/4 15/8 45/32
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Figure 10. Just Intonation Compared to Pythagorean Tuning

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                    Just Intonation

   6/5  9/5    4/3  1  3/2 9/8   5/3  5/4 15/8 45/32  25/24 25/16
    |    |      |   |   |   |     |    |    |    |      |     |
    Eb---Bb--k--F---C---G---D--k--A----E----B----F#--k--C#----G#

 Eb---Bb----F--C--G---D----A-----E------B-------F#------C#--------G#
  |    |    |  |  |   |    |     |      |       |       |          |
32/27 16/9 4/3 1 3/2 9/8 27/16 81/64 243/128 729/512 2187/2048 6561/4096

                   Pythagorean Tuning
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Figure 11. The Four Modes of Just Intonation

Nt	Pythagorean	Just (G)	Just (C)	Just (F)	Just (Bb)
B#	531441/524288	125/64 (-3k)	125/64 (-3k)	125/64 (-3k)	125/64 (-3k)
E#	177147/131072	675/512 (-2k)	125/96 (-3k)	125/96 (-3k)	125/96 (-3k)
A#	59049/32768	225/128 (-2k)	225/128 (-2k)	125/72 (-3k)	125/72 (-3k)
D#	19683/16384	75/64 (-2k)	75/64 (-2k)	75/64 (-2k)	125/108 (-3k)
G#	6561/4096	25/16 (-2k)	25/16 (-2k)	25/16 (-2k)	25/16 (-2k)
C#	2187/2048	135/128 (-1k)	25/24 (-2k)	25/24 (-2k)	25/24 (-2k)
F#	729/512	45/32 (-1k)	45/32 (-1k)	25/18 (-2k)	25/18 (-2k)
B	243/128	15/8 (-1k)	15/8 (-1k)	15/8 (-1k)	50/27 (-2k)
E	81/64	5/4 (-1k)	5/4 (-1k)	5/4 (-1k)	5/4 (-1k)
A	27/16	27/16	5/3 (-1k)	5/3 (-1k)	5/3 (-1k)
D	9/8	9/8	9/8	10/9 (-1k)	10/9 (-1k)
G	3/2	3/2	3/2	3/2	40/27 (-1k)
C	1	1	1	1	1
F	4/3	27/20 (+1k)	4/3	4/3	4/3
Bb	16/9	9/5 (+1k)	9/5 (+1k)	16/9	16/9
Eb	32/27	6/5 (+1k)	6/5 (+1k)	6/5 (+1k)	32/27
Ab	128/81	8/5 (+1k)	8/5 (+1k)	8/5 (+1k)	8/5 (+1k)
Db	256/243	27/25 (+2k)	16/15 (+1k)	16/15 (+1k)	16/15 (+1k)
Gb	1024/729	36/25 (+2k)	36/25 (+2k)	64/45 (+1k)	64/45 (+1k)
Cb	2048/2187	48/25 (+2k)	48/25 (+2k)	48/25 (+2k)	256/135 (+1k)
Fb	8192/6561	32/25 (+2k)	32/25 (+2k)	32/25 (+2k)	32/25 (+2k)
Bbb	32768/19683	216/125 (+3k)	128/75 (+2k)	128/75 (+2k)	128/75 (+2k)
Ebb	65536/59049	144/125 (+3k)	144/125 (+3k)	256/125 (+2k)	256/125 (+2k)
Abb	262144/177147	192/125 (+3k)	192/125 (+3k)	192/125 (+3k)	1024/675 (+2k)
Dbb	524288/531441	128/125 (+3k)	128/125 (+3k)	128/125 (+3k)	128/125 (+3k)

Figure 12. Schematic of Quarter-Comma Meantone Temperament

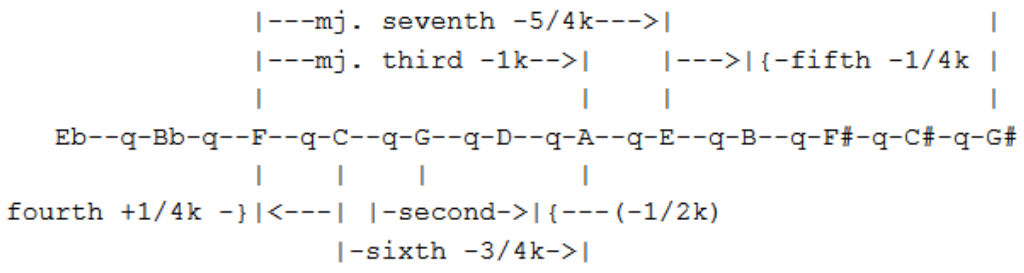

Figure 13. Quarter-Comma Meantone Temperament

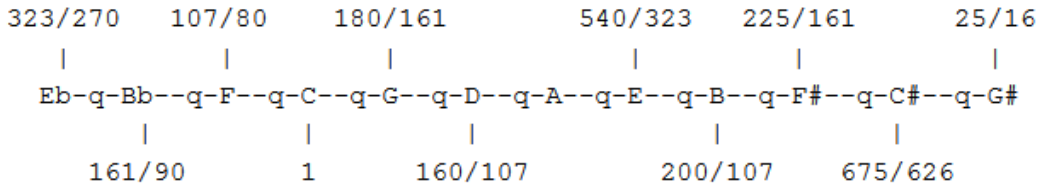

Figure 14. Schematic of Well Temperament

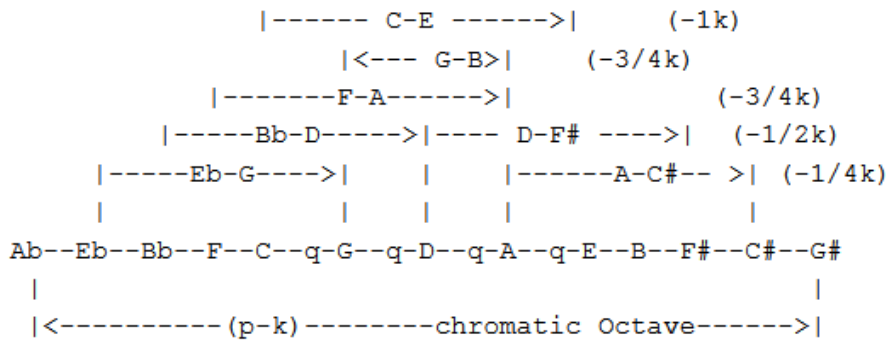

Figure 15. Well Temperament

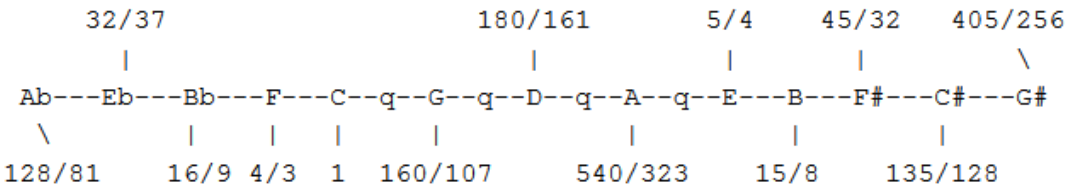

Figure 16. Schismatic and Syntonic Equal Temperament

Tuning	Note	Schismatic ET	Syntonic ET
1.000009	Dbb	C - k + 11sk	
1.498295	Abb	G - k + 10sk	
1.22454	Ebb	D - k + 9sk	
1.683681	Bbb	A - k + 8sk	
1.261336	Fb	E - k + 7sk	
1.887739	Cb	B - k + 6sk	
1.414207	Gb	F# - k + 5sk	
1.059459	Db	C# - k + 4sk	F######### - 5k
1.587396	Ab	G# - k + 3sk	D####### - 4k
1.189205	Eb	D# - k + 2sk	B##### - 3k
1.781795	Bb	A# - k + 1sk	G### - 2k
1.334839	F	E# - k + 0sk	E# - 1k
1.0	C	C	
1.498308	G	Abb + k	Abb + 1k
1.22464	D	Ebb + k - 1sk	Fbbb + 2k
1.681797	A	Bbb + k - 2sk	Dbbbbb + 3k
1.259925	E	Fb + k - 3sk	Bbbbbbbb + 4k
1.887756	B	Cb + k - 4sk	Gbbbbbbbbb + 5k
1.41422	F#	Gb + k - 5sk	Ebbbbbbbbbb + 6k
1.059468	C#	Db + k - 6sk	
1.58741	G#	Ab + k - 7sk	
1.189215	D#	Eb + k - 8sk	
1.781811	A#	Bb + k - 9sk	
1.334851	E#	F + k - 10sk	
1.000009	B#	C + k - 11sk	

Figure 17. Just (C-Major mode) Schismatic and Syntonic Equal Temperaments

Tuning	Note	Just Schis. ET	Just Synt. ET
1.059459	Db	C# + 1k + 4sk	F######### + 9k
1.587396	Ab	G# + 1k + 3sk	D####### + 7k
1.189205	Eb	D# + 1k + 2sk	B##### + 5k
1.781795	Bb	A# + 1k + 1sk	G### + 3k
1.334839	F	E# + 2k	E# + 2k
1.0	C		C
1.498308	G	Abb - 2k	Abb - 2k
1.22464	D	Ebb - 2k - 1sk	Fbbb - 3k
1.681797	A	Bbb - 1k - 2sk	Dbbbbb - 5k
1.259925	E	Fb - 1k - 3sk	Bbbbbbbb - 7k
1.887756	B	Cb - 1k - 4sk	Gbbbbbbbb - 9k
1.41422	F#	Gb - 1k - 5sk	Ebbbbbbbbbb - 10k